

Supplemental Appendix to “Strategic (Dis)Integration”

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E Proofs of Supporting Results

E.1 Technical Lemmas

We now state and prove a few technical lemmas used to prove other supporting results.

First, we show the household problem may be split into an intratemporal problem plus a condition on prices.

Lemma 3. *For each country i , the conditions*

$$\begin{aligned} \{c_{it}\}_{t=1,2} &\in \arg \max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \\ \text{s.t.} \quad &\sum_{t=1,2} (p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} l_{i1}) - \mathbb{1}_{i=H} [(\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w \tau_t] \cdot m_{Ht}) \leq 0 \\ c_{it} + \mathbb{1}_{t=1} l_{i1} &= y_{it} + m_{it} \\ p_t^w \cdot m_{it} &= 0 \\ p_{Ht} &= (\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w (1 + \tau_t) \end{aligned}$$

are equivalent to the condition that there exists $\eta_i > 0$ for which

$$\begin{aligned} c_{it} &\in \arg \max_c u_{it}(c) \quad \text{s.t.} \quad p_{it} \cdot c \leq p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} l_{i1}) + p_{it} \cdot m_{it} \\ p_{it} &= \eta_i u_{it,c}(c_{it}) \\ c_{it} + \mathbb{1}_{t=1} l_{i1} &= y_{it} + m_{it} \\ p_t^w \cdot m_{it} &= 0 \\ p_{Ht} &= (\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w (1 + \tau_t). \end{aligned}$$

Proof. We prove the forward and reverse directions in turn.

Forward direction: The goods market clearing condition implies

$$p_{it} \cdot c_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) + p_{it} \cdot m_{it}.$$

Since the provided household optimization condition implies household optimization within each period, we have

$$c_{it} \in \arg \max_c u_{it}(c) \quad \text{s.t.} \quad p_{it} \cdot c \leq p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) + p_{it} \cdot m_{it}.$$

The provided household optimization condition also implies the existence of inverse Lagrange multipliers $\eta_i > 0$ for which

$$p_{it} = \eta_i u_{it,c}(c_{it}).$$

Reverse direction: The provided trade balance and non-arbitrage conditions ensure that

$$p_{Ht} \cdot m_{Ht} = [(\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w \tau_t] \cdot m_{Ht}.$$

The budget constraints of the intratemporal household problems and local nonsatiation therefore imply

$$\sum_{t=1,2} (p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) - \mathbb{1}_{i=H} [(\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w \tau_t] \cdot m_{Ht}) = 0.$$

Combining this observation with the condition on prices and the concavity of household utility implies that, by Lagrangian sufficiency,

$$\begin{aligned} \{c_{it}\}_{t=1,2} &\in \arg \max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \\ \text{s.t.} \quad &\sum_{t=1,2} (p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) - \mathbb{1}_{i=H} [(\mathcal{R}_F / \mathcal{R}_H)^{t-1} p_t^w \tau_t] \cdot m_{Ht}) \leq 0. \end{aligned}$$

□

Second, we show that, taking trade and capital as given, the domestic equilibrium conditions are equivalent to domestic efficiency.

Lemma 4. *Given any time-1 net imports m_{i1} and capital k_i , a time-1 profile $\{c_{i1}, y_{i1}, \iota_{i1}, p_{i1}, r_{i1}\}$*

satisfies the trade-and-capital-conditional intratemporal equilibrium conditions

$$\begin{aligned}
c_{i1} &\in \arg \max_c u_{i1}(c) \quad s.t. \quad p_{i1} \cdot c \leq p_{i1} \cdot (y_{i1} - \iota_{i1}) + p_{i1} \cdot m_{i1} \\
\{y_{i1}, k_{i1}^y\} &\in \arg \max_{y, k^y} p_{i1} \cdot y - r_{i1} \cdot k^y \quad s.t. \quad G_{i1}(y, k^y) \leq 0 \\
\iota_{i1} &\in \arg \max_{\iota_1} (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}) \cdot k_i - p_{i1} \cdot \iota_1 \quad s.t. \quad \Lambda_{i1}(k_i, \iota_1) \leq 0 \\
c_{i1} + \iota_{i1} &= y_{i1} + m_{i1} \\
k_{i1}^y &= k_i
\end{aligned}$$

if and only if there exists $\eta_{i1} > 0$ for which

$$\begin{aligned}
\{y_{i1}, \iota_{i1}\} &\in \arg \max_{y, \iota} u_{i1}(y + m_{i1} - \iota) \quad s.t. \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\
p_{i1} &= \eta_{i1} V_{i1,m}(m_{i1}, k_i) \\
r_{i1} &= -G_{i1,k}(y_{i1}, k_i) / G_{i1,y_g^*}(y_{i1}, k_i) \times p_{i1g^*} \\
c_{i1} + \iota_{i1} &= y_{i1} + m_{i1} \\
k_{i1}^y &= k_i.
\end{aligned}$$

Moreover, either set of conditions implies

$$u_{i1}(c_{i1}) = V_{i1}(m_{i1}, k_i) \quad \text{and} \quad u_{i1,c}(c_{i1}) = V_{i1,m}(m_{i1}, k_i).$$

Similarly, given any time-2 net imports m_{i2} and capital k_i , a time-2 profile $\{c_{i2}, y_{i2}, p_{i2}, r_{i2}\}$ satisfies the trade-and-capital-conditional intratemporal equilibrium conditions

$$\begin{aligned}
c_{i2} &\in \arg \max_c u_{i2}(c) \quad s.t. \quad p_{i2} \cdot c \leq p_{i2} \cdot y_{i2} + p_{i2} \cdot m_{i2} \\
\{y_{i2}, k_{i2}^y\} &\in \arg \max_{y, k^y} p_{i2} \cdot y - r_{i2} \cdot k^y \quad s.t. \quad G_{i2}(y, k^y) \leq 0 \\
c_{i2} &= y_{i2} + m_{i2} \\
k_{i2}^y &= k_i
\end{aligned}$$

if and only if there exists $\eta_{i2} > 0$ for which

$$\begin{aligned} y_{i2} &\in \arg \max_y u_{i2}(y + m_{i2}) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0 \\ p_{i2} &= \eta_{i2} V_{i2,m}(m_{i2}, k_i) \\ r_{i2} &= -G_{i2,k}(y_{i2}, k_i) / G_{i2,y_{g^*}}(y_{i2}, k_i) \times p_{i2g^*} \\ c_{i2} &= y_{i2} + m_{i2} \\ k_{i2}^y &= k_i. \end{aligned}$$

Moreover, either set of conditions implies

$$u_{i2}(c_{i2}) = V_{i2}(m_{i2}, k_i) \quad \text{and} \quad u_{i2,c}(c_{i2}) = V_{i2,m}(m_{i2}, k_i).$$

Proof. We prove the time-1 case. The time-2 case is analogous.

Forward direction: The provided time-1 conditions in each country i are the same as the equilibrium conditions in a closed-economy model with endowment m_{i1} and two competitive firms—one corresponding to goods production and one corresponding to producing a single feasible level of capital output (i.e., k_i) at least cost. The first welfare theorem applies to each of these as-if economies. So production and investment are socially efficient, i.e.,

$$\{y_{i1}, \iota_{i1}\} \in \arg \max_{y, \iota} u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0. \quad (\text{E.1})$$

Next, the provided household optimization condition implies there exists $\eta_{i1} > 0$ for which

$$p_{i1} = \eta_{i1} u_{i1,c}(c_{i1}).$$

Moreover, the definition of $V_{i1}(\cdot, \cdot)$ implies $u_{i1,c}(c) = V_{i1,m}(m_{i1}, k_i)$ (and $u_{i1}(c) = V_{i1}(m_{i1}, k_i)$) if $c = y_{i1}^* + m_{i1} - \iota_{i1}^*$ for y_{i1}^* and ι_{i1}^* the output and investment levels associated with $V_{i1}(\cdot, \cdot)$'s definition. The provided goods market clearing condition and (E.1) imply this is the case for $c = c_{i1}$, so

$$p_{i1} = \eta_{i1} V_{i1,m}(m_{i1}, k_i).$$

Finally, the desired condition for r_{i1} follows from first-order conditions of the provided optimality condition for goods producers.

Reverse direction: Consider the same closed economies discussed in the forward direction. The provided conditions for $\{y_{i1}, \iota_{i1}\}$, m_{i1} , and k_{i1}^y imply that the allocation is both technologically feasible and efficient. In each as-if economy, the second welfare theorem

applies. So there exist some prices p_{i1}^* and r_{i1}^* for which

$$\begin{aligned} c_{i1} &\in \arg \max_c u_{i1}(c) \quad \text{s.t.} \quad p_{i1}^* \cdot c \leq p_{i1}^* \cdot (y_{i1} - \iota_{i1}) + p_{i1}^* \cdot m_{i1} \\ \{y_{i1}, k_{i1}^y\} &\in \arg \max_{y, k^y} p_{i1}^* \cdot y - r_{i1}^* \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0 \\ \iota_{i1} &\in \arg \max_{\iota_1} (r_{i1}^* (1 + \mathbb{1}_{i=H} s_1) + r_{i2}^*) \cdot k_i - p_{i1}^* \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota_1) \leq 0. \end{aligned} \quad (\text{E.2})$$

It remains to show $p_{i1}^* = p_{i1}$ and $r_{i1}^* = r_{i1}$, up to a price constant. To see this, note that the implied first-order condition for households implies that for some $\eta_{i1}^* > 0$

$$p_{i1}^* = \eta_{i1}^* u_{i1,c}(c_{i1}).$$

Moreover, the definition of $V_{i1}(\cdot, \cdot)$ implies $u_{i1,c}(c) = V_{i1,m}(m_{i1}, k_i)$ (and $u_{i1}(c) = V_{i1}(m_{i1}, k_i)$) if $c = y_{i1}^* + m_{i1} - \iota_{i1}^*$ for y_{i1}^* and ι_{i1}^* the output and investment levels associated with $V_{i1}(\cdot, \cdot)$'s definition. The provided conditions for y_{i1} and ι_{i1} and goods market clearing imply this is the case for $c = c_{i1}$, so

$$p_{i1}^* = \eta_{i1}^* V_{i1,m}(m_{i1}, k_i) = \frac{\eta_{i1}^*}{\eta_{i1}} p_{i1}.$$

Similarly, the implied first-order condition for production firms implies

$$r_{i1}^* = -G_{i1,k}(y_{i1}, k_i) / G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*}^*,$$

so we have $r_{i1}^* = \frac{\eta_{i1}^*}{\eta_{i1}} r_{i1}$. We conclude that (E.2) holds when p_{i1}^* and r_{i1}^* are replaced with p_{i1} and r_{i1} , respectively. $\square$

Third, we provide a simple characterization of the optimality conditions for capital in each country.

Lemma 5. *Suppose that for some $\eta_i > 0$,*

$$\begin{aligned} p_{i1} &= \eta_i V_{i1,m}(m_{i1}, k_i) \\ \tilde{p}_{i2}(a) &= \eta_i V_{i2,m}(\tilde{m}_{i2}(a), k_i) \\ r_{i1} &= -G_{i1,k}(y_{i1}, k_i) / G_{i1,y_{g^*}}(y_{i1}, k_i) \times p_{i1g^*} \\ \tilde{r}_{i2}(a) &= -G_{i2,k}(\tilde{y}_{i2}(a), k_i) / G_{i2,y_{g^*}}(\tilde{y}_{i2}(a), k_i) \times \tilde{p}_{i2g^*}(a) \\ \{y_{i1}, \iota_{i1}\} &\in \arg \max_{y, \iota} u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\ \tilde{y}_{i2}(a) &\in \arg \max_y u_{i2}(y + \tilde{m}_{i2}(a)) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0. \end{aligned}$$

Then i 's capital k_i satisfies

$$k_i \in \arg \max_k \max_{\iota_1} (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \quad (\text{E.3})$$

if and only if

$$\eta_i [V_{i1,k}(m_{i1}, k_i) + V_{i2,k}(\tilde{m}_{i2}(a), k_i)] = -\mathbb{1}_{i=H} r_{i1} s_1. \quad (\text{E.4})$$

Moreover, (E.4) holds for $i = F$ if and only if

$$k_F \in \arg \max_k V_{F1}(m_{F1}, k) + V_{F2}(\tilde{m}_{F2}(a), k).$$

Proof. The final equivalence in the lemma statement is immediate from our convexity assumptions. It therefore suffices to show that (E.3) and (E.4) are equivalent.

Using the definition of $V_{it}(\cdot, \cdot)$ and the given conditions for y_{i1} , ι_{i1} , and $\tilde{y}_{i2}(a)$, (E.4) is equivalent to the statement that there exists $\varphi^* \geq 0$ for which

$$\begin{aligned} & -\frac{\eta_i V_{i1,m_{g^*}}(m_{i1}, k_i)}{G_{i1,y_{g^*}}(y_{i1}, k_i)} G_{i1,k}(y_{i1}, k_i) - \eta_i \varphi^* \Lambda_{i1,k}(k_i, \iota_{i1}) \\ & -\frac{\eta_i V_{i2,m_{g^*}}(\tilde{m}_{i2}(a), k_i)}{G_{i2,y_{g^*}}(\tilde{y}_{i2}(a), k_i)} G_{i2,k}(\tilde{y}_{i2}(a), k_i) = -\mathbb{1}_{i=H} r_{i1} s_1 \\ & V_{i1,m}(m_{i1}, k_i) = -\varphi^* \Lambda_{i1,\iota}(k_i, \iota_{i1}). \end{aligned} \quad (\text{E.5})$$

At the same time, using the given conditions for prices, (E.3) is equivalent to the statement that there exist some $\varphi \geq 0$ and $\hat{\iota}_{i1}$ for which

$$\begin{aligned} & -\frac{\eta_i V_{i1,m_{g^*}}(m_{i1}, k_i)}{G_{i1,y_{g^*}}(y_{i1}, k_i)} G_{i1,k}(y_{i1}, k_i) - \eta_i \varphi \Lambda_{i1,k}(k_i, \hat{\iota}_{i1}) \\ & -\frac{\eta_i V_{i2,m_{g^*}}(\tilde{m}_{i2}(a), k_i)}{G_{i2,y_{g^*}}(\tilde{y}_{i2}(a), k_i)} G_{i2,k}(\tilde{y}_{i2}(a), k_i) = -\mathbb{1}_{i=H} r_{i1} s_1 \\ & V_{i1,m}(m_{i1}, k_i) = -\varphi \Lambda_{i1,\iota}(k_i, \hat{\iota}_{i1}) \\ & \Lambda_{i1}(k_i, \hat{\iota}_{i1}) = 0, \end{aligned} \quad (\text{E.6})$$

where the condition that $\Lambda_{i1}(k_i, \hat{\iota}_{i1}) = 0$ is by nonsatiation. The final two conditions and our convexity assumptions imply

$$\hat{\iota}_{i1} \in \arg \max_{\iota} u_{i1}(y_{i1} + m_{i1} - \iota) \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota) \leq 0,$$

which ensures $\hat{\iota}_{i1} = \iota_{i1}$. The conditions (E.5) and (E.6) are therefore equivalent when $\varphi = \varphi^*$.

This completes the proof. $\square$

E.2 Proof of Lemma 1

To begin, note that by Lemma 3, the planner's problem as stated in Appendix A.2 is equivalent to the problem of choosing a profile $\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k_{it}^y, k_i, p_t^w, p_{it}, r_{it}, \mathcal{R}_i, a, s_1, \tau_1, \tau_2, \tilde{\tau}_2(\cdot), \eta_i\}$ that solves the problem in (A.1) after replacing the condition for $\{c_{it}\}_{t=1,2}$ with

$$\begin{aligned} c_{it} &\in \arg \max_c u_{it}(c) \quad \text{s.t.} \quad p_{it} \cdot c \leq p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) + p_{it} \cdot m_{it} \\ p_{it} &= \eta_i u_{it,c}(c_{it}). \end{aligned}$$

Next, recall $\tilde{c}_{F2}(\tau_2, k_H, k_F)$ is the unique level of consumption consistent with the time-2 equilibrium conditions given trade taxes τ_2 and capital stocks k_H and k_F , or

$$\begin{aligned} c_{i2} &\in \arg \max_c u_{i2}(c) \quad \text{s.t.} \quad p_{i2} \cdot c \leq p_{i2} \cdot y_{i2} + p_{i2} \cdot m_{i2} \\ \{y_{i2}, k_{i2}^y\} &\in \arg \max_{y, k^y} p_{i2} \cdot y - r_{i2} \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \leq 0 \\ c_{i2} &= y_{i2} + m_{i2}, \quad m_{H2} + m_{F2} = 0, \quad \text{and} \quad k_i = k_{i2}^y \\ p_2^w \cdot m_{i2} &= 0 \\ p_{F2} &= p_2^w \quad \text{and} \quad p_{H2} = (\mathcal{R}_F / \mathcal{R}_H) p_2^w (1 + \tau_2), \end{aligned}$$

where note we have used trade balance to replace trade tax revenues $\mathbb{1}_{i=H}(\mathcal{R}_F / \mathcal{R}_H)(p_2^w \tau_2) \cdot m_{i2}$ with the domestic value of net imports, $p_{i2} \cdot m_{i2}$, in the household budget constraint. We may therefore replace $\tilde{c}_{F2}(\tilde{\tau}_2(\hat{a}), k_H, k_F)$ in the equilibrium conditions with a generic function $\tilde{c}_{F2}(\hat{a})$ provided that we additionally require there to exist functions $\tilde{c}_{i2}(\cdot)$, $\tilde{y}_{i2}(\cdot)$, $\tilde{m}_{i2}(\cdot)$, $\tilde{k}_{i2}^y(\cdot)$, $\tilde{p}_{i2}(\cdot)$, $\tilde{r}_{i2}(\cdot)$, and $\tilde{p}_2^w(\cdot)$ that satisfy the time-2 equilibrium conditions given trade taxes $\tilde{\tau}_2(\cdot)$ and capital stocks k_H and k_F . And since the time-2 equilibrium conditions are invariant to a price constant, it is without loss to additionally impose that $\tilde{p}_{i2}(a)$ is normalized the same way as the actual on-path prices p_{i2} . In other words, the planner's problem is equivalent to choosing a profile

$$\left\{ c_{it}, y_{it}, \iota_{i1}, m_{it}, k_{it}^y, k_i, p_t^w, p_{it}, r_{it}, \mathcal{R}_i, a, s_1, \tau_1, \tau_2, \tilde{\tau}_2(\cdot), \eta_i, \tilde{c}_{i2}(\cdot), \tilde{y}_{i2}(\cdot), \tilde{m}_{i2}(\cdot), \tilde{k}_{i2}^y(\cdot), \tilde{p}_{i2}(\cdot), \tilde{r}_{i2}(\cdot), \tilde{p}_2^w(\cdot) \right\}$$

that maximizes the objective

$$u_{H1}(c_{H1}) + u_{H2}(\tilde{c}_{H2}(a)) + z_H(a)$$

subject to

$$\begin{aligned}
c_{i1} &\in \arg \max_c u_{i1}(c) \quad \text{s.t.} \quad p_{i1} \cdot c \leq p_{i1} \cdot (y_{i1} - \iota_{i1}) + p_{i1} \cdot m_{i1} \\
\tilde{c}_{i2}(\hat{a}) &\in \arg \max_c u_{i2}(c) \quad \text{s.t.} \quad \tilde{p}_{i2}(\hat{a}) \cdot c \leq \tilde{p}_{i2}(\hat{a}) \cdot \tilde{y}_{i2}(\hat{a}) + \tilde{p}_{i2}(\hat{a}) \cdot \tilde{m}_{i2}(\hat{a}) \\
p_{i1} &= \eta_i u_{i1,c}(c_{i1}), \quad \tilde{p}_{i2}(a) = \eta_i u_{i2,c}(\tilde{c}_{i2}(a)) \\
\{y_{i1}, k_{i1}^y\} &\in \arg \max_{y, k^y} p_{i1} \cdot y - r_{i1} \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0 \\
\{\tilde{y}_{i2}(\hat{a}), \tilde{k}_{i2}^y(\hat{a})\} &\in \arg \max_{y, k^y} \tilde{p}_{i2}(\hat{a}) \cdot y - \tilde{r}_{i2}(\hat{a}) \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \leq 0 \\
\{k_i, \iota_{i1}\} &\in \arg \max_{k, \iota_1} (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \\
c_{i1} + \iota_{i1} &= y_{i1} + m_{i1}, \quad m_{H1} + m_{F1} = 0, \quad \text{and} \quad k_i = k_{i1}^y \\
\tilde{c}_{i2}(\hat{a}) &= \tilde{y}_{i2}(\hat{a}) + \tilde{m}_{i2}(\hat{a}), \quad \tilde{m}_{H2}(\hat{a}) + \tilde{m}_{F2}(\hat{a}) = 0, \quad \text{and} \quad k_i = \tilde{k}_{i2}^y(\hat{a}) \\
p_1^w \cdot m_{i1} &= 0 \\
\tilde{p}_2^w(\hat{a}) \cdot \tilde{m}_{i2}(\hat{a}) &= 0 \\
p_{F1} &= p_1^w \quad \text{and} \quad p_{H1} = p_1^w(1 + \tau_1) \\
\tilde{p}_{F2}(\hat{a}) &= \tilde{p}_2^w(\hat{a}) \quad \text{and} \quad \tilde{p}_{H2}(\hat{a}) = (\mathcal{R}_F/\mathcal{R}_H)(1 + \tilde{\tau}_2(\hat{a}))\tilde{p}_2^w(\hat{a}) \\
u_{F2}(\tilde{c}_{F2}(a)) + z_F(a) &\geq u_{F2}(\tilde{c}_{F2}(\neg a)) + z_F(\neg a) \\
\tau_2 &= \tilde{\tau}_2(a) \\
\tilde{\tau}_2(\cdot) &\in \mathcal{T}_2(k_H, k_F) \\
c_{i2} &= \tilde{c}_{i2}(a), \quad y_{i2} = \tilde{y}_{i2}(a), \quad m_{i2} = \tilde{m}_{i2}(a), \quad k_{i2}^y = \tilde{k}_{i2}^y(a) \\
p_{i2} &= \tilde{p}_{i2}(a), \quad r_{i2} = \tilde{r}_{i2}(a), \quad p_2^w = \tilde{p}_2^w(a).
\end{aligned}$$

By the convexity of $\Lambda_{i1}(\cdot, \cdot)$, the condition above for $\{k_i, \iota_{i1}\}$ is equivalent to the two conditions

$$\begin{aligned}
\iota_{i1} &\in \arg \max_{\iota_1} [r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(a)] \cdot k_i - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota_1) \leq 0 \\
k_i &\in \arg \max_k \max_{\iota_1} [r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(a)] \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0.
\end{aligned}$$

Lemma 4 then implies this version of the planner's problem is, in turn, equivalent to the condition that for some η_{i1} and $\tilde{\eta}_{i2}(\cdot)$, the same profile maximizes

$$V_{H1}(m_{H1}, k_H) + V_{H2}(\tilde{m}_{H2}(a), k_H) + z_H(a)$$

subject to

$$\begin{aligned}
\{y_{i1}, \iota_{i1}\} &\in \arg \max_{y, \iota} u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\
\tilde{y}_{i2}(\hat{a}) &\in \arg \max_y u_{i2}(y + \tilde{m}_{i2}(\hat{a})) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0 \\
p_{i1} &= \eta_i V_{i1,m}(m_{i1}, k_i), \quad \tilde{p}_{i2}(a) = \eta_i V_{i2,m}(\tilde{m}_{i2}(a), k_i) \\
p_{i1} &= \eta_{i1} V_{i1,m}(m_{i1}, k_i), \quad \tilde{p}_{i2}(\hat{a}) = \tilde{\eta}_{i2}(\hat{a}) V_{i2,m}(\tilde{m}_{i2}(\hat{a}), k_i) \\
r_{i1} &= -G_{i1,k}(y_{i1}, k_i) / G_{i1,y_{i1}}(y_{i1}, k_i) \times p_{i1g^*} \\
\tilde{r}_{i2}(\hat{a}) &= -G_{i2,k}(\tilde{y}_{i2}(\hat{a}), k_i) / G_{i2,y_{i2}}(\tilde{y}_{i2}(\hat{a}), k_i) \times \tilde{p}_{i2g^*}(\hat{a}) \\
k_i &\in \arg \max_k \max_{\iota_1} (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \\
c_{i1} + \iota_{i1} &= y_{i1} + m_{i1}, \quad m_{H1} + m_{F1} = 0, \quad \text{and} \quad k_i = k_{i1}^y \\
\tilde{c}_{i2}(\hat{a}) &= \tilde{y}_{i2}(\hat{a}) + \tilde{m}_{i2}(\hat{a}), \quad \tilde{m}_{H2}(\hat{a}) + \tilde{m}_{F2}(\hat{a}) = 0, \quad \text{and} \quad k_i = \tilde{k}_{i2}^y(\hat{a}) \\
V_{F1,m}(m_{F1}, k_F) \cdot m_{F1} &= 0 \\
V_{F2,m}(\tilde{m}_{F2}(\hat{a}), k_F) \cdot \tilde{m}_{F2}(\hat{a}) &= 0 \\
p_{F1} &= p_1^w \quad \text{and} \quad p_{H1} = p_1^w(1 + \tau_1) \\
\tilde{p}_{F2}(\hat{a}) &= \tilde{p}_2^w(\hat{a}) \quad \text{and} \quad \tilde{p}_{H2}(\hat{a}) = (\mathcal{R}_F / \mathcal{R}_H)(1 + \tilde{\tau}_2(\hat{a})) \tilde{p}_2^w(\hat{a}) \\
V_{F2}(\tilde{m}_{F2}(a), k_F) - V_{F2}(\tilde{m}_{F2}(-a), k_F) &\geq z_F(-a) - z_F(a) \\
\tau_2 &= \tilde{\tau}_2(a) \\
\tilde{\tau}_2(\cdot) &\in \mathcal{T}_2(k_H, k_F) \\
c_{i2} &= \tilde{c}_{i2}(a), \quad y_{i2} = \tilde{y}_{i2}(a), \quad m_{i2} = \tilde{m}_{i2}(a), \quad k_{i2}^y = \tilde{k}_{i2}^y(a) \\
p_{i2} &= \tilde{p}_{i2}(a), \quad r_{i2} = \tilde{r}_{i2}(a), \quad p_2^w = \tilde{p}_2^w(a).
\end{aligned}$$

Note that we may equivalently remove the clause “for some η_{i1} and $\tilde{\eta}_{i2}(\cdot)$ ” and replace

$$\begin{aligned}
p_{i1} &= \eta_i V_{i1,m}(m_{i1}, k_i), \quad \tilde{p}_{i2}(a) = \eta_i V_{i2,m}(\tilde{m}_{i2}(a), k_i) \\
p_{i1} &= \eta_{i1} V_{i1,m}(m_{i1}, k_i), \quad \tilde{p}_{i2}(\hat{a}) = \tilde{\eta}_{i2}(\hat{a}) V_{i2,m}(\tilde{m}_{i2}(\hat{a}), k_i)
\end{aligned}$$

with¹

$$p_{i1} = \eta_i V_{i1,m}(m_{i1}, k_i) \quad \text{and} \quad \tilde{p}_{i2}(\hat{a}) = \eta_i V_{i2,m}(\tilde{m}_{i2}(\hat{a}), k_i).$$

Given international goods market clearing, we can freely replace m_{H1} with $-m_{F1}$ and $\tilde{m}_{H2}(\hat{a})$ with $-\tilde{m}_{F2}(\hat{a})$. Given the relationships between world prices, domestic prices, and

¹Here we implicitly use the observation that $\mathcal{T}_2(k_H, k_F)$ is closed under Lerner transformations under both Assumption 1 and Assumption 2. That is, if $\tilde{\tau}_2(\cdot) \in \mathcal{T}_2(k_H, k_F)$ and for some scalar-valued function $\varphi(\cdot)$ we have $1 + \tilde{\tau}_2(\hat{a}) = \varphi(\hat{a})[1 + \tilde{\tau}_2(\hat{a})]$ for all $\hat{a}$, then $\hat{\tau}_2(\cdot) \in \mathcal{T}_2(k_H, k_F)$.

Meade utility functions, we can also replace $\tilde{\tau}_2(\cdot) \in \mathcal{T}_2(k_H, k_F)$ with

$$\chi \frac{V_{H2,m}(\tilde{m}_{H2}(\cdot), k_H)}{V_{F2,m}(\tilde{m}_{F2}(\cdot), k_F)} - 1 \in \mathcal{T}_2(k_H, k_F) \quad \text{and} \quad \chi = \frac{\eta_H \mathcal{R}_H}{\eta_F \mathcal{R}_F},$$

where χ is an additional choice variable for the planner. By Lemma 5 and Assumption 6, we can replace the condition for k_F with

$$k_F = \tilde{k}_F(m_{F1}, \tilde{m}_{F2}(a))$$

and we can replace the condition for k_H with

$$\eta_H [V_{H1,k}(m_{H1}, k_H) + V_{H2,k}(\tilde{m}_{H2}(a), k_H)] = -r_{H1}s_1.$$

Employing these observations and rearranging conditions, we arrive at the desired version of the planner's problem from the statement of the lemma.

E.3 Proof of Lemma 2

Recall that

$$u_{H2}^*(k_H, k_F) \equiv \max_{\tau_2} u_{H2}(\tilde{c}_{H2}(\tau_2, k_H, k_F)),$$

where $\tilde{c}_{H2}(\tau_2, k_H, k_F)$ is time-2 consumption in the unique solution (up to a price constant) to the time-2 equilibrium conditions. These conditions are:

$$\begin{aligned} c_{i2} &\in \arg \max_{c_2} u_{i2}(c_2) \quad \text{s.t.} \quad P_{i2} \cdot c_2 \leq P_{i2} \cdot y_{i2} + \mathbf{1}_{i=H}(P_2^w \tau_2) \cdot m_{i2} \\ \{y_{i2}, k_{i2}^y\} &\in \arg \max_{y, k^y} P_{i2} \cdot y - R_{i2} \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \leq 0 \\ c_{i2} &= y_{i2} + m_{i2}, \quad m_{H2} + m_{F2} = 0 \quad \text{and} \quad k_i = k_{i2}^y \\ P_2^w \cdot m_{i2} &= 0 \\ P_{F2} &= P_2^w \quad \text{and} \quad P_{H2} = (1 + \tau_2)P_2^w. \end{aligned}$$

First, we argue that the right-hand side of (A.2) is an upper bound on $u_{H2}^*(k_H, k_F)$. This follows from two observations: First, conditional on Foreign net imports m_{F2} , the market clearing condition $m_{H2} = -m_{F2}$ and the definition of Meade utility imply that Home's consumption utility is at most $V_{H2}(-m_{F2}, k_H)$. Second, by Lemma 4, any trade taxes τ_2 result in world prices proportional to Foreign's marginal Meade utility $V_{F2,m}(m_{F2}, k_F)$. So since any equilibrium must satisfy trade balance, the constraint on the right-hand side of (A.2) must hold under any Home trade taxes. Since the objective in (A.2) is at least as high

as Home's consumption utility and the constraint is a subset of the full time-2 equilibrium conditions, the right-hand side provides an upper bound.

Second, we argue that the right-hand side of (A.2) is a lower bound on $u_{H2}^*(k_H, k_F)$. To do so, it suffices to, given any solution m_{F2} to (A.2), construct a time-2 equilibrium profile $\{c_{i2}, y_{i2}, k_{i2}^y, m_{i2}, P_2^w, P_{i2}, R_{i2}, \tau_2\}$ given capital $\{k_i\}$ that delivers Home economic utility $V_{H2}(-m_{F2}, k_H)$. Starting from m_{F2} , construct the remainder of the profile as follows:

$$\begin{aligned} m_{H2} &\equiv -m_{F2} \\ y_{i2} &\equiv \arg \max_y u_{i2}(y + m_{i2}) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0 \\ c_{i2} &\equiv y_{i2} + m_{i2} \quad \text{and} \quad k_{i2}^y = k_i \\ P_{i2} &\equiv u_{i2,c}(c_{i2}) \quad \text{and} \quad R_{i2} \equiv \frac{P_{i2g}^*}{-G_{i2,yg^*}(y_{i2}, k_i)} G_{i2,k}(y_{i2}, k_i) \\ P_2^w &\equiv P_{F2} \quad \text{and} \quad \tau_{2g} \equiv P_{H2g}/P_{F2g} - 1. \end{aligned}$$

This profile satisfies all time-2 equilibrium conditions by construction, given the regularity conditions imposed in Appendix A.1. Since y_{H2} is set to solve Home's Meade problem, the profile also delivers Home time-2 economic utility equal to $V_{H2}(m_{H2}, k_H) = V_{H2}(-m_{F2}, k_H)$, as desired.

F Microfoundation of the Credibility Constraint

This appendix provides a microfoundation for Home's credibility constraint, Assumption 2. We do this by embedding the baseline model of Section 2 into a repeated geopolitical game between Home and a new Foreign country each period. We then find Home trade threats that are implementable in a subgame perfect equilibrium.

F.1 Environment

There are infinitely many periods $T \in \{0, 1, \dots\}$. Within each period, Home, a new Foreign, and atomistic investors play a stage game similar to the model described in Section 2. Home discounts future periods at a rate $\delta \in (0, 1)$.

Stage game. Within a period T , the timing is the same as in Section 2 with the added wrinkle that Home can renege on its trade threats after Foreign chooses its geopolitical action.

- i. Home starts by choosing time-1 capital subsidies s_1^T and trade taxes τ_1^T as well as a time-2 trade tax threat $\tilde{\tau}_2^T(\cdot)$ that maps geopolitical actions to trade tax vectors.

- ii. Atomistic firms choose capital k_i^T and investment ι_{i1}^T in each country i subject to the feasibility constraint that $\Lambda_{i1}(k_i^T, \iota_{i1}^T) \leq 0$.
- iii. Foreign chooses a geopolitical action $a^T \in \{\underline{a}, \bar{a}\}$.
- iv. Home chooses a trade tax vector τ_2^T that may or may not equal the realization of its threat at Foreign's action, $\tilde{\tau}_2^T(a^T)$.

Payoffs. Let $\mathcal{P}^T = \{s_1^T, \tau_1^T, \tau_2^T, \{k_i^T, \iota_{i1}^T\}\}$ represent a period- T profile of Home's realized policies s_1^T , τ_1^T , and τ_2^T , and private capital and investment choices $\{k_i^T, \iota_{i1}^T\}$. We assume that for each $\mathcal{P}^T$ there exists a unique equilibrium profile (up to price indeterminacy) consistent with household and firm optimization, cost-minimizing investment, market clearing, trade balance in each period, and the necessary relationship between world and local prices; we denote the corresponding value of each equilibrium variable x as $\tilde{x}(\mathcal{P}^T)$. We moreover assume that time-1 consumption in each country only depends on the time-1 subset of the profile, $\mathcal{P}_1^T \equiv \mathcal{P}^T \setminus \{\tau_2^T\}$.

Stage-game payoffs are as follows: The payoff of each country i is

$$U_i^T = u_{i1}[\tilde{c}_{i1}^T(\mathcal{P}_1^T)] + u_{i2}[\tilde{c}_{i2}^T(\mathcal{P}^T)] + z_i(a^T).$$

Firms are atomistic and so do not internalize that their capital investments affect prices. They therefore behave as if their payoffs are

$$[\bar{r}_{i1}^T(1 + \mathbb{1}_{i=H}s_1^T) + \bar{r}_{i2}^T] \cdot k_i^T - \bar{p}_{i1}^T \cdot \iota_{i1}^T$$

while taking $\bar{r}_{it}^T \equiv \tilde{r}_{it}(\mathcal{P}^T)$ and $\bar{p}_{i1}^T \equiv \tilde{p}_{i1}(\mathcal{P}^T)$ as given.

Since there is a new Foreign in each period and since firms are atomistic, only their stage-game payoffs matter. Since moreover Foreign acts after $\mathcal{P}_1^T$ has been determined, it acts solely on the basis of its time-2 utility,

$$u_{F2}[\tilde{c}_{F2}^T(\mathcal{P}^T)] + z_F(a^T).$$

Since Home is a strategic, repeat player, it acts based on its lifetime payoffs. These are

$$\mathcal{U}_H = \sum_{T=0}^{\infty} \delta^T U_H^T.$$

F.2 Home and Foreign Strategies

In this section, we describe a grim-trigger strategy profile in the repeated game under which Home's behavior in each stage game is consistent with the reduced-form constraint imposed by Assumption 2. We subsequently argue this strategy profile is a subgame perfect equilibrium.

At a high level, the strategy is simple to state: On the equilibrium path, everyone knows that Home will always carry out its trade threats. However, if Home fails to carry out a threat, it loses its credibility; everyone knows that Home will act myopically in all future periods, reevaluating its time-2 trade taxes after Foreign chooses its action and setting them to whatever delivers the highest time-2 economic payoff in that period given capital. Understanding this, Foreign's geopolitical behavior off the equilibrium path ignores Home's threats. Finally, on the equilibrium path, Home constrains its trade threats to ensure that its temptation to renege never exceeds the continuation value of maintaining its honest reputation (i.e., of remaining on the equilibrium path in future periods).

To describe the strategy profile formally, it is useful to partition the set of all histories into two pieces. We let $\mathbf{h}^{ON}$ denote the set of histories h at any period T along which Home has carried out all previous threats (i.e., $\tau_2^{T'} = \tilde{\tau}_2^{T'}(a^{T'})$ for all $T' < T$). We let $\mathbf{h}^{OFF}$ denote the complementary set of histories along which Home has ever not followed through on a threat.

We begin by describing Home's strategy in terms of a scalar Δ^T specified below. Conditional on Δ^T , we describe the strategy profile in reverse order on the decision nodes within each period. We begin with Home's decision of how to react to Foreign's action a^T at a period- T history h during which the Home and market actions $s_1^T, \tau_1^T, \tilde{\tau}_2^T(\cdot), \{k_i^T, \iota_{i1}^T\}$ have already been chosen. The strategy profile prescribes that Home will carry out its threat unless doing so has a time-2 economic cost of more than Δ^T relative to its best alternative. Formally, Home's behavior is given by a (history-dependent) function $\hat{\tau}_2^T(\cdot)$ defined by

$$\hat{\tau}_2^T(a^T) = \begin{cases} \tilde{\tau}_2^T(a^T) & \text{if } u_{H2}[\tilde{c}_{H2}(\mathcal{P}_1^T \cup \{\tilde{\tau}_2^T(a^T)\})] + \Delta^T \geq u_{H2}^*(k_H^T, k_F^T) \\ \tau_2^{*T} & \text{else} \end{cases},$$

where $u_{H2}^*(k_H^T, k_F^T) = \max_{\tau_2} u_{H2}(\tilde{c}_{H2}(\tau_2, k_H^T, k_F^T))$ and where τ_2^{*T} is the associated maximizer.

We next consider decision nodes at which Foreign must choose which action to take, given the history including Home and market actions $s_1^T, \tau_1^T, \tilde{\tau}_2^T(\cdot), \{k_i^T, \iota_{i1}^T\}$ in the same period. Letting $\hat{\tau}_2^T(\cdot)$ be the history-dependent function defined above, our strategy prescribes that Foreign chooses whichever action maximizes its time-2 utility under the belief that $\hat{\tau}_2^T(\cdot)$

indeed describes Home's subsequent reaction, i.e.,

$$a^T \in \arg \max_a u_{F2} \left[\tilde{c}_{F2} \left(\mathcal{P}_1^T \cup \{\hat{\tau}_2^T(a)\} \right) \right] + z_F(a).$$

Next, consider decision nodes at which private firms must choose capital and investment, given the history including Home policy announcements $s_1^T, \tau_1^T, \tilde{\tau}_2^T(\cdot)$ in the same period. The strategy prescribes that, for some prices $\bar{p}_{i1}^T$ and $\bar{r}_{it}^T$, these firms choose k_i^T and ι_{i1}^T that solve

$$\max_{k, \iota_1} \left[\bar{r}_{i1}^T (1 + \mathbb{1}_{i=H} s_1^T) + \bar{r}_{i2}^T \right] \cdot k - \bar{p}_{i1}^T \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0.$$

The prices $\bar{p}_{i1}^T$ and $\bar{r}_{it}^T$ at which these actions are chosen are those consistent with the resulting profile:

$$\bar{p}_{i1}^T = \tilde{p}_{i1} \left(s_1^T, \tau_1^T, \hat{\tau}_2^T(a^T), \{k_i^T, \iota_{i1}^T\} \right), \quad \text{and} \quad \bar{r}_{it}^T = \tilde{r}_{it} \left(s_1^T, \tau_1^T, \hat{\tau}_2^T(a^T), \{k_i^T, \iota_{i1}^T\} \right),$$

where we note that $\hat{\tau}_2^T(\cdot)$ and a^T (defined as above) are implicitly functions of $\{k_i^T, \iota_{i1}^T\}$ whose dependence we suppress for brevity. We assume a unique such fixed point $\{k_i^T, \iota_{i1}^T\}$ exists for any Home policy commitments $s_1^T, \tau_1^T, \tilde{\tau}_2^T(\cdot)$ and any prior history.

Finally, we must specify the strategy's prescriptions for Home's policy commitments. We begin by describing these commitments up to a parameter Δ^T . We then describe how Δ^T depends on the history.

For a given value of Δ^T , Home's strategy is to choose policies that maximize its stage payoff conditional on how firms and Foreign will subsequently respond and subject to the constraint that Home must, at the final decision node of the period, not prefer the economic consequences of reneging on its threat by more than Δ^T under either Foreign geopolitical action. Formally, Home's actions s_1^T, τ_1^T , and $\tilde{\tau}_2^T(\cdot)$ must solve

$$\begin{aligned} & \max_{s_1, \tau_1, \tilde{\tau}_2(\cdot)} \max_{a, \{k_i, \iota_{i1}\}} u_{H1}[\tilde{c}_{H1}(\mathcal{P} \setminus \{\tilde{\tau}_2(a)\})] + u_{H2}[\tilde{c}_{H2}(\mathcal{P})] + z_H(a) \\ & \text{s.t.} \quad \mathcal{P} = \{s_1, \tau_1, \tilde{\tau}_2(a), \{k_i, \iota_{i1}\}\}, \quad \mathcal{P}' = \{s_1, \tau_1, \tilde{\tau}_2(\neg a), \{k_i, \iota_{i1}\}\} \\ & \quad \{k_i, \iota_{i1}\} \in \arg \max_{k, \iota_1} [\tilde{r}_{i1}(\mathcal{P})(1 + \mathbb{1}_{i=H} s_1) + \tilde{r}_{i2}(\mathcal{P})] \cdot k - \tilde{p}_{i1}(\mathcal{P}) \cdot \iota_1 \\ & \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \\ & \quad u_{F2}[\tilde{c}_{F2}(\mathcal{P})] + z_F(a) \geq u_{F2}[\tilde{c}_{F2}(\mathcal{P}')] + z_F(\neg a) \\ & \quad u_{H2}[\tilde{c}_{H2}(\mathcal{P})] + \Delta^T \geq u_{H2}^*(k_H, k_F) \\ & \quad u_{H2}[\tilde{c}_{H2}(\mathcal{P}')] + \Delta^T \geq u_{H2}^*(k_H, k_F). \end{aligned} \tag{F.1}$$

We denote the value of this problem by $U_H(\Delta^T)$. Home's strategy at off-path histories

$h \in \mathbf{h}^{OFF}$ is to choose policies that solve (F.1) at $\Delta^T = 0$. Home's strategy at on-path histories $h \in \mathbf{h}^{ON}$ is to choose policies that solve (F.1) at $\Delta^T = \Delta^*$ defined implicitly as the highest fixed point of the equation

$$\frac{\delta}{1-\delta} [U_H(\Delta^*) - U_H(0)] = \Delta^*.$$

We assume such a fixed point exists.

F.3 Subgame Perfect Equilibrium

We now argue that the strategy described above constitutes a subgame perfect equilibrium.

Subgame perfection is immediate for private firms and for Foreign. Private firms simply maximize profits taking as given rational expectations about prices. Foreign chooses the utility-maximizing action given correct beliefs about Home's response to its actions.

It remains to verify that Home's prescribed behavior is optimal at each decision node. First, consider decision nodes at which Home chooses whether to carry out its threat or to renege. At off-path histories $h \in \mathbf{h}^{OFF}$, Home simply chooses the trade taxes that maximize its static payoff; this is rational because once we have left the equilibrium path, Home's continuation value does not depend on its behavior. At on-path histories $h \in \mathbf{h}^{ON}$, Home's continuation value is $\delta U_H(\Delta^*)/(1-\delta)$ if it carries out its threat and $\delta U_H(0)/(1-\delta)$ otherwise. So it is rational for Home to carry out its threat if and only if its time-2 economic preference for choosing the ex-post optimal trade taxes does not exceed

$$\frac{\delta}{1-\delta} U_H(\Delta^*) - \frac{\delta}{1-\delta} U_H(0) = \Delta^*,$$

where the equality is by the construction of Δ^* . This is precisely how we have defined Home's behavior.

Finally, consider decision nodes at which Home chooses its policy commitments. At off-path histories $h \in \mathbf{h}^{OFF}$, the fact that Home's policies solve (F.1) at $\Delta^T = 0$ implies by construction that they maximize Home's stage-game payoff conditional on how subsequent actors will respond. Since at such histories Home's continuation value is independent of its behavior, the actions are therefore optimal. At on-path histories $h \in \mathbf{h}^{ON}$, the argument involves one additional step: First, note that Home can always do weakly better by choosing policy commitments that it is willing to carry out under either Foreign action honestly later in the period. Otherwise, Home could do at least as well by unconditionally offering the time-2 trade taxes it will end up choosing anyway (since its unresponsiveness to Foreign's true action will be anticipated and this way it at least maintains credibility). Given that Home

chooses policy commitments that it will carry out under either Foreign action, the fact that Home’s policies solve (F.1) implies by construction that they maximize Home’s stage-game payoff conditional on how subsequent actors will respond. Since they induce Home to carry out its threat later in the period, they also achieve the highest possible continuation value and so are dynamically optimal.

F.4 Connection to Reduced-Form Constraint

Along the equilibrium path of the subgame perfect equilibrium described above, Home’s policy commitments solve the baseline planner’s problem subject to an additional constraint: Home’s time-2 economic utility must, under all Foreign geopolitical actions, be not more than Δ^* below the highest time-2 economic utility it could achieve under any time-2 trade taxes, given capital. This is precisely what Assumption 2 imposes.

G Data Appendix

G.1 Data Sources

We now describe the data sources used in our calibration.

World Input-Output Database. We obtain data on value added and labor compensation from the Socio Economic Accounts data in the 2016 release of the World Input-Output Database (WIOD). We use only values from the most recent year for which data is available, 2014. This release of WIOD makes data available for a set $\mathcal{I}_{2014}^{WIOD}$ of 43 countries and a set $\mathcal{N}_{2014}^{WIOD}$ of 56 sectors. For each such country i and sector n , we obtain a measure of value added VA_{in}^{WIOD} and labor payments W_{in}^{WIOD} .

OECD Inter-Country Input-Output tables. We obtain data on flows of intermediate and final goods across countries and sectors from the OECD’s Inter-Country Input-Output tables (ICIO), as cleaned by Adão et al. (2024). We use data from only a single year, 2017. We choose this year because it is the last year before the imposition of the 2018 “Trump tariffs” and so arguably provides a more accurate “steady state” representation of the world trading system. The ICIO makes data available for a set $\mathcal{I}^{ICIO+}$ of 76 countries plus a rest-of-the-world aggregate and a set $\mathcal{N}^{ICIO+}$ of 45 sectors. Adão et al. (2024) merge Belgium and Luxembourg, resulting in a set $\mathcal{I}^{ICIO}$ of 75 countries plus a rest-of-the-world aggregate, and they merge the pharmaceutical and chemical sectors, resulting in a set $\mathcal{N}^{ICIO}$ of 44 sectors.

We obtain three variables from the ICIO data. First, for each pair of countries i, j and sector n , we obtain a measure $C_{ijn}^{w,ICIO}$ of final consumption expenditures by households, governments, and non-profit institutions serving households in j on goods in sector n originating in i . Second, for each pair of countries i and j and each sector n , we obtain a measure $Inv_{ijn}^{w,ICIO}$ of investment expenditures in j on goods in sector n originating in i . We ignore changes in inventories and direct purchases abroad. Third, for each pair of countries i, j and each pair of sectors n, n' , we obtain a measure of the intermediate expenditures $Int_{ijn'}^{w,ICIO}$ by firms in country j and sector n' on goods in sector n originating in i . All of these measures are at world prices (FOB).

Cleaned CEPII BACI trade data. We obtain data on bilateral flows of HS6 goods between countries in 2017 from CEPII’s BACI data set, as cleaned by [Adão et al. \(2024\)](#). The BACI makes bilateral trade flow data available for a set $\mathcal{H}^{BACI}$ of 5113 HS6 products between a set of 183 countries. The cleaned data that we borrow from [Adão et al. \(2024\)](#) aggregates these flows across countries to the level of $\mathcal{I}^{ICIO}$. We thereby obtain, for every pair of countries $i, j \in \mathcal{I}^{ICIO}$ and every HS6 product $h \in \mathcal{H}$, a measure of bilateral trade flows $X_{ijh}^{w,BACI}$ at world prices (FOB). These flows only cover merchandise goods, and they omit “domestic trade,” i.e., $X_{iih}^{w,BACI} = 0$.

Cleaned WITS tariff data. We obtain data on bilateral tariffs in 2017 from the World Bank’s World Integrated Trade Solution (WITS), as cleaned by [Adão et al. \(2024\)](#). These authors follow a cleaning procedure similar to that proposed by [Teti \(2023\)](#) to address substantial imputation errors in the data provided by the WITS database. They also augment this data from USITC and MacMap with additional information on discriminatory tariffs applied by a subset of countries. The cleaned data that we borrow from [Adão et al. \(2024\)](#) provides, for all country pairs i, j in $\mathcal{I}^{ICIO}$ and products h in the set of HS6 products $\mathcal{H}$, a measure $\tau_{ijh}^{m,ABCD}$ of the bilateral, statutory, ad-valorem-equivalent tariffs that j applies to imports of h from i .

Imputed capital network data. We borrow estimates of countries’ cross-sectoral investment networks from [Ding \(2021\)](#). [Ding \(2021\)](#) draws on a range of data sets to compute these networks in 1997 for a set $\mathcal{I}^{Ding}$ of 40 countries plus a rest-of-the-world aggregate and a set $\mathcal{N}^{Ding}$ of 27 sectors. For all countries i in $\mathcal{I}^{Ding}$ and all sector pairs n, n' in $\mathcal{N}^{Ding}$, we obtain a measure $Inv_{in'n}^{w,Ding}$ of the investment expenditures of sector n in country i on goods produced by sector n' (from all origins, in FOB world prices), in 1997.

We also use the rates of sector-specific capital depreciation δ_n^{Ding} and the risk-free interest

rate ($r^{f,Ding} = 0.03$) provided in [Ding \(2021\)](#).

Crosswalks. We hand-code several crosswalks from country and sector classifications in the raw data to their finest common coarsenings. For countries we use a set $\mathcal{I}$ of 39 countries plus a rest-of-the-world aggregate; this is the same as $\mathcal{I}^{Ding}$ except that it merges Belgium and Luxembourg. For sectors we use a set $\mathcal{N}$ of 26 sectors; this is the same as $\mathcal{N}^{Ding}$ except that it merges textile and leather manufacturing. We let $D_{\mathcal{I}}^{ICIO}$, $D_{\mathcal{I}}^{WIOD}$, $D_{\mathcal{I}}^{Ding}$, $D_{\mathcal{N}}^{ICIO}$, and $D_{\mathcal{N}}^{WIOD}$ denote the many-to-one crosswalk matrices that encode this coarsening. For example, $[D_{\mathcal{I}}^{ICIO}]_{ij}$ is an indicator for whether country or country-group i contains country or country-group j . Additionally, we borrow from [Adão et al. \(2024\)](#) a many-to-one crosswalk $D_{\mathcal{N}\mathcal{H}}^{ICIO}$ that assigns each HS6 product $h \in \mathcal{H}$ to a unique sector $n \in \mathcal{N}^{ICIO}$. Concretely, its (n, h) value is an indicator for whether sector n contains product h .

G.2 Data Cleaning

We now describe the cleaning procedure we apply to the raw data described in Supplemental Appendix [G.1](#) in order to arrive at the statistics we use for calibration. These are measures of sector-level tariffs $\tau_{ijn}^{m,data}$ and export subsidies $\tau_{ijn}^{x,data}$; shares of country-sector trade flows directed toward each origin-sector, ζ_{ijn}^{data} ; shares of gross output directed toward labor, $\theta_{in}^{\ell,data}$, capital, $\theta_{in}^{k,data}$, and intermediates from each sector, $\theta_{in'n}^{Q,data}$; shares of capital expenditures directed toward investment in capital goods from each sector, $\theta_{in'n}^{\iota,data}$; shares of each country's consumption expenditures directed toward goods from each sector, $\theta_{jn}^{C,data}$; and each country's final consumption expenditures, C_i^{data} .

Sectoral tariff revenues. As a preliminary step, we compute the implied bilateral tariff revenues collected within each ICIO sector, rescaling by the ratio of ICIO- to BACI-measured bilateral sectoral trade. We set

$$T_{ijn}^{imp} \equiv \frac{X_{ijn}^{w,ICIO}}{\sum_{h \in \mathcal{H}} [D_{\mathcal{N}\mathcal{H}}^{ICIO}]_{nh} X_{ijh}^{w,BACI}} \sum_{h \in \mathcal{H}} [D_{\mathcal{N}\mathcal{H}}^{ICIO}]_{nh} \tau_{ijh}^{m,ABCD} X_{ijh}^{w,BACI}$$

where $X_{ijn}^{w,ICIO} \equiv C_{ijn}^{w,ICIO} + Inv_{ijn}^{w,ICIO} + \sum_{n' \in \mathcal{N}^{ICIO}} Int_{ijn'}^{w,ICIO}$.

We set $T_{ijn}^{imp} = 0$ for any (i, j, n) with no BACI-reported trade, i.e., where the denominator above is zero. This includes all service sectors and all domestic trade (i.e., $i = j$).

Aggregation. We begin by aggregating all of the raw data sets to their finest common coarsenings, i.e., $\mathcal{I}$ for countries and $\mathcal{N}$ for sectors. We denote the aggregated versions

of the measures described in Supplemental Appendix G.1 by $\bar{V}A_{in}^{WIOD}$, $\bar{W}_{in}^{WIOD}$, $\bar{C}_{ijn}^{w,ICIO}$, $\bar{Inv}_{ijn}^{w,ICIO}$, $\bar{Int}_{ijn'}^{w,ICIO}$, $\bar{X}_{ijh}^{w,BACI}$, and $\bar{Inv}_{in'n}^{w,Ding}$; we also aggregate T_{ijn}^{imp} to a measure $\bar{T}_{ijn}^{imp}$.

As textiles and leather manufacturing have the same depreciation rate according to δ_n^{Ding} , we construct the $|\mathcal{N}|$ -length vector $\bar{\delta}_n^{Ding}$ by simply setting this common rate as the depreciation rate for their merged sector.

Sectoral average trade taxes. We set sectoral import tariffs $\tau_{ijn}^{m,data}$ as the ratio of bilateral tariff revenues in a sector, computed above, to pre-tariff trade flows:

$$\tau_{ijn}^{m,data} = \bar{T}_{ijn}^{imp} / X_{ijn}^{w,imp}$$

where $X_{ijn}^{w,imp} \equiv \bar{C}_{ijn}^{w,ICIO} + \bar{Inv}_{ijn}^{w,ICIO} + \sum_{n' \in \mathcal{N}} \bar{Int}_{ijn'}^{w,ICIO}$.

We set $\tau_{ijn}^{m,data} = 0$ for any (i, j, n) with no ICIO-reported trade, i.e., where the denominator above is zero.

We assume that export subsidies are zero, i.e.,²

$$\tau_{ijn}^{x,data} \equiv 0.$$

Trade shares. We next use the adjusted trade flows computed above to compute trade shares across sectors and origins. We begin by computing aggregate tariff-inclusive flows implied by the ICIO data:

$$X_{ijn}^{imp} \equiv (1 + \tau_{ijn}^{m,data}) X_{ijn}^{w,imp}.$$

We then compute, for each origin country i , destination country j , and sector n , the share of j 's expenditures in n that are directed to i , i.e.,

$$\zeta_{ijn}^{data} \equiv X_{ijn}^{imp} / \sum_{o \in \mathcal{I}} X_{ojn}^{imp}.$$

Tariff-inclusive expenditures. We next adjust ICIO consumption, investment, and intermediate spending to account for tariffs. Specifically, we assume that any country's expenditures on goods from a given sector—whether they are consumption, investment, or intermediate expenditures—are sourced from origin countries in the same proportions as one another. Under this assumption, the trade flows computed above indicate how each spending

²Our approach can be easily extended to accommodate non-zero export subsidies, given data on them.

type must be adjusted to account for tariffs. Concretely, we set

$$\begin{aligned} C_{jn}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{C}_{ojn}^{w,ICIO} \\ Inv_{jn}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{Inv}_{ojn}^{w,ICIO} \\ Int_{jnn'}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{Int}_{onjn'}^{w,ICIO}. \end{aligned}$$

Intermediate and labor shares of gross output. We next compute the share of each country-sector's revenues (i.e., gross output) it spends on intermediate goods from each sector. Note that intermediate expenditures include tariff payments, while revenues do not. We set

$$\theta_{in'n}^{Q,data} \equiv \frac{Int_{in'n}^{imp}}{\sum_{j \in \mathcal{I}} X_{ijn}^{w,imp}}.$$

We winsorize the sum of $\theta_{in'n}^{Q,data}$ across supplying industries n' at 0.99, rescaling proportionally within (i, n) pairs. This winsorization affects 52 of 1040 total country-sector pairs.

We next compute labor shares of gross output in each country and sector. We begin by computing the labor share in each country and sector based on our aggregated WIOD data:

$$LS_{in}^{data} \equiv \bar{W}_{in}^{WIOD} / \bar{V} A_{in}^{WIOD}.$$

We winsorize labor shares at 0.01 and 0.99; this affects the computed labor shares of 42 out of 1040 total country-sector pairs. In the one case where $\bar{V} A_{in}^{WIOD} = 0$ (WIOD reports value added of zero in the Mexican real estate sector), we set LS_{in}^{data} equal to the global aggregate labor share in sector n , i.e.,

$$LS_{in}^{data} \equiv \sum_{j \in \mathcal{I}} \bar{W}_{jn}^{WIOD} / \sum_{j \in \mathcal{I}} \bar{V} A_{jn}^{WIOD}.$$

We then compute labor shares of gross output at the country-sector level by combining intermediate shares of gross output with labor shares of value added:

$$\theta_{in}^{\ell,data} \equiv LS_{in}^{data} \left(1 - \sum_{n' \in \mathcal{N}} \theta_{in'n}^{Q,data} \right).$$

Level and composition of consumption. We compute total final consumption in each country as the sum of sectoral consumption:

$$C_i^{data} \equiv \sum_{n \in \mathcal{N}} C_{in}^{imp}.$$

We compute the share of each sector in each country's consumption basket:

$$\theta_{in}^{C,data} \equiv \frac{C_{in}^{imp}}{C_i^{data}}.$$

Directed capital shares. We next adapt the methodology of [Ding \(2021\)](#) to compute the share of each country-sector pair's capital expenditures that it directs toward goods from each sector.

First, for each country $i \in \mathcal{I}$ and each pair of sectors $n, n' \in \mathcal{N}$, we impute the investment expenditures—of each sector n in each country i on goods in each sector n' —implied by country i 's total investment in n' (as computed above) and sector n 's contribution to country i 's investment in n' from [Ding \(2021\)](#). That is,

$$Inv_{in'n}^{imp} \equiv \frac{Inv_{in'n}^{w,Ding}}{\sum_{n'' \in \mathcal{N}} Inv_{in'n''}^{w,Ding}} Inv_{in'}^{imp}.$$

Second, we compute the profits required to rationalize these investments. We do so using the risk-free rate, capital-sector-specific depreciation rates, and the assumption that the economy is in steady state. Concretely, we set

$$\Pi_{in'n}^{imp} \equiv \frac{r^{f,Ding}}{\delta_{n'}^{Ding}} Inv_{in'n}^{imp}.$$

Third and finally, we compute the share of total payments to capital, at the country-sector level, directed toward investment in each sector. That is, we set

$$\theta_{in'n}^{\iota,data} \equiv \frac{Inv_{in'n}^{imp}}{\sum_{n'' \in \mathcal{N}} (Inv_{in'n''}^{imp} + \Pi_{in'n''}^{imp})}.$$

H Quantitative Appendix

In this appendix, we describe the algorithm by which we compute the optimal peacetime tariffs. Supplemental Appendix [H.1](#) shows that, due to our focus on the limit in which the conflict period is short and geopolitical conflict is limited, the optimal policies in our quantitative

model can be computed as the solution of three static maximization problems. Supplemental Appendix H.2 provides an outline for the computational algorithm. Supplemental Appendix H.3 describes how we build the geopolitical components of the optimal peacetime taxes out of the solutions to those problems. Supplemental Appendix H.4 includes detailed descriptions of the equilibrium solvers used. Supplemental Appendix H.5 uses equilibrium equations developed in Supplemental Appendix H.4 to describe how we calibrate the quantitative model.

H.1 Decomposition of the Maximization Problem

This section, as well as Supplemental Appendices H.2 and H.3, maintains notation consistent with the theoretical framework from Section 2 and its extension to bystander countries in Appendix C.4. Supplemental Appendix H.4 switches to the notation of the quantitative model, with Supplemental Appendix H.4.1 providing the explicit mapping between notations.

To study the limit in which geopolitical conflict has little impact on Home's planning problem, we also introduce a variable φ that indexes both countries' geopolitical preferences. That is, country i 's geopolitical preference for action a is $z_i(a; \varphi)$. Similarly, to accommodate the $\beta \rightarrow 0$ limit in our general framework, where β does not appear as a parameter, we define

$$V_{it}^\circ(m_{it}, k_i) \equiv V_{it}(m_{it}, k_i)/\beta^{t-1}, \quad z_i^\circ(a; \varphi) \equiv z_i(a; \varphi)/\beta, \quad \text{and} \quad \Delta^\circ \equiv \Delta/\beta.$$

We will consider limits as $\beta \rightarrow 0$ while $V_{it}^\circ(\cdot, \cdot)$ and Δ° remain fixed and $z_i^\circ(\cdot)$ varies only due to concurrent changes in φ . It will also be useful to introduce notation for net imports and welfare. Noting that, in our quantitative model, trade taxes and capital uniquely determine the allocation within any time, we let

$$m_{it}^\circ(\tau_t, k) \quad \text{and} \quad W_{it}^\circ(\tau_t, k) \equiv V_{it}^\circ[m_{it}^\circ(\tau_t, k), k_i]$$

denote the associated levels of net imports and welfare in each country, where $k \equiv \{k_i\}$.

Using this notation, we can write Home's planning problem, conditional on implementing action a , as

$$\begin{aligned} \max_{\tau_1, \tau_2, \tau_2^{OFF}, k} \quad & W_{H1}^\circ(\tau_1, k) + \beta [W_{H2}^\circ(\tau_2, k) + z_H^\circ(a; \varphi)] \\ \text{s.t.} \quad & 0 = V_{i1, k}^\circ(m_{i1}^\circ(\tau_1, k), k_i) + \beta V_{i2, k}^\circ(m_{i2}^\circ(\tau_2, k), k_i), \forall i \neq H \\ & \beta W_{F2}^\circ(\tau_2, k) + \beta z_F^\circ(a; \varphi) \geq \beta W_{F2}^\circ(\tau_2^{OFF}, k) + \beta z_F^\circ(-a; \varphi) \\ & \min [\beta W_{H2}^\circ(\tau_2, k), \beta W_{H2}^\circ(\tau_2^{OFF}, k)] + \beta \Delta^\circ \geq \beta W_{H2}^\circ(\tau_2^*(k), k), \end{aligned} \tag{H.1}$$

where $\tau_2^*(k)$ are the trade taxes that maximize Home utility at time 2, taking capital as given,

or

$$\tau_2^*(k) \in \arg \max_{\tau_2} W_{H2}^\circ(\tau_2, k). \quad (\text{H.2})$$

We denote the solution to (H.1) by $\tau_1(\beta, \varphi)$, $\tau_2(\beta, \varphi)$, $\tau_2^{OFF}(\beta, \varphi)$, and $k(\beta, \varphi)$, and we denote the Lagrange multipliers on the capital constraints, geopolitical IC constraint, and credibility constraint by $\lambda_i^k(\beta, \varphi)$, $\lambda^{IC}(\beta, \varphi)$, and $\lambda^\Delta(\beta, \varphi)$, respectively. This is a complicated dynamic maximization problem featuring 3 separate static equilibria: time 1, on-path time 2, and off-path time 2. We propose to instead solve the following maximization problems:

- i. The time-1 terms-of-trade maximization problem,

$$\begin{aligned} \max_{\tau_1, k} W_{H1}^\circ(\tau_1, k) \\ \text{s.t. } 0 = V_{i1, k}^\circ(m_{i1}^\circ(\tau_1, k), k_i), \forall i \neq H. \end{aligned} \quad (\text{H.3})$$

We denote the solution by $\dot{\tau}_1$ and $\dot{k}$.

- ii. The time-2, on-path problem,

$$\max_{\tau_2} W_{H2}^\circ(\tau_2, \dot{k}) - \sum_{i \neq H} \lambda_i^k \cdot V_{i2, k}^\circ(m_{i2}^\circ(\tau_2, \dot{k}), \dot{k}_i), \quad (\text{H.4})$$

where λ_i^k are some weights on foreign rental rates. We denote the solution by $\dot{\tau}_2$.

- iii. The time-2, off-path sanctioning problem,

$$\max_{\tau_2^{OFF}} W_{H2}^\circ(\tau_2^{OFF}, \dot{k}) + \lambda^{OFF} W_{F2}^\circ(\tau_2^{OFF}, \dot{k}), \quad (\text{H.5})$$

where λ^{OFF} (which may be negative) is some weight put on foreign utility. We denote the solution by $\dot{\tau}_2^{OFF}$.

The following proposition shows that the original problem (H.1) is equivalent to (H.3), (H.4), and (H.5) for properly chosen λ_i^k and λ^{OFF} in a limit that corresponds to our quantitative exercise.³

Proposition 5. *Suppose that the solutions to (H.1), (H.3), (H.4), and (H.5) are uniquely characterized by their Lagrangian first-order conditions. Further suppose that in the limit as*

³Recall that, in the quantitative exercise, we take time 2 to be short and we take geopolitical preferences to differ just enough that Home starts to use peacetime policies. Note that the latter limit implies the geopolitical IC constraint and the credibility constraint have vanishing multipliers. (The fact that the credibility constraint's multiplier is vanishing relies on the assumption that Home has a large enough continuation value Δ° to carry out its dynamically optimal time-2 trade policy, which differs from its ex-post optimal time-2 trade policy. Our calibration satisfies this property.)

$\beta \rightarrow 0$ and $\varphi \rightarrow \hat{\varphi}$, $\lambda^{IC}(\beta, \varphi), \lambda^\Delta(\beta, \varphi) > 0$ and $\lambda^{IC}(\beta, \varphi), \lambda^\Delta(\beta, \varphi) \rightarrow 0$. Suppose also that for all β and φ , Home's utility in the solution to (H.1) is lower off-path than on-path. Then there exist $\dot{\lambda}_i^k$ and λ^{OFF} such that $\tau_1(\beta, \varphi) \rightarrow \dot{\tau}_1$, $\tau_2(\beta, \varphi) \rightarrow \dot{\tau}_2$, $\tau_2^{OFF}(\beta, \varphi) \rightarrow \dot{\tau}_2^{OFF}$, and $k(\beta, \varphi) \rightarrow \dot{k}$ as $\beta \rightarrow 0$ and $\varphi \rightarrow \hat{\varphi}$.

Proof. The solutions to (H.3)-(H.5), i.e., $\dot{\tau}_1$, $\dot{\tau}_2$, $\dot{\tau}_2^{OFF}$, and $\dot{k}$, are characterized by the first-order conditions

$$\begin{aligned} 0 &= W_{H1, \tau_1}^\circ - \sum_{i \neq H} \dot{\lambda}_i^k \frac{dV_{i1, k}^\circ}{d\tau_1}, & 0 &= W_{H1, k}^\circ - \sum_{i \neq H} \dot{\lambda}_i^k \frac{dV_{i1, k}^\circ}{dk}, \\ 0 &= W_{H2, \tau_2}^\circ - \sum_{i \neq H} \lambda_i^k \cdot \frac{dV_{i2, k}^\circ}{d\tau_2}, & 0 &= W_{H2, \tau_2}^{\circ, OFF} + \lambda^{OFF} W_{F2, \tau_2}^{\circ, OFF}, \end{aligned}$$

along with the restrictions

$$0 = V_{i1, k}^\circ, \forall i \neq H,$$

where $\dot{\lambda}_i^k$ are the Lagrange multipliers on the capital investment restrictions and where all derivatives above are evaluated at the associated solutions to their respective problems.

Similarly, the solution to (H.1), i.e., $\tau_1(\beta, \varphi)$, $\tau_2(\beta, \varphi)$, $\tau_2^{OFF}(\beta, \varphi)$, and $k(\beta, \varphi)$, satisfies

$$\begin{aligned} 0 &= W_{H1, \tau_1}^\circ(\beta, \varphi) - \sum_{i \neq H} \lambda_i^k(\beta, \varphi) \cdot \frac{dV_{i1, k}^\circ(\beta, \varphi)}{d\tau_1} \\ 0 &= W_{H1, k}^\circ(\beta, \varphi) + \beta W_{H2, k}^\circ(\beta, \varphi) - \sum_{i \neq H} \lambda_i^k(\beta, \varphi) \cdot \left(\frac{dV_{i1, k}^\circ(\beta, \varphi)}{dk} + \beta \frac{dV_{i2, k}^\circ(\beta, \varphi)}{dk} \right) \\ &\quad + \lambda^{IC}(\beta, \varphi) \beta \left(W_{F2, k}^\circ(\beta, \varphi) - W_{F2, k}^{\circ, OFF}(\beta, \varphi) \right) - \lambda^\Delta(\beta, \varphi) \beta \left(W_{H2, k}^{\circ, *}(\beta, \varphi) - W_{H2, k}^{\circ, OFF}(\beta, \varphi) \right) \\ &\quad - \lambda^\Delta(\beta, \varphi) \beta W_{H2, \tau_2}^{\circ, *}(\beta, \varphi) \tau_{2, k}^*(\beta, \varphi) \\ 0 &= W_{H2, \tau_2}^\circ(\beta, \varphi) - \sum_{i \neq H} \lambda_i^k(\beta, \varphi) \cdot \frac{dV_{i2, k}^\circ(\beta, \varphi)}{d\tau_2} + \lambda^{IC}(\beta, \varphi) \beta W_{F2, \tau_2}^\circ(\beta, \varphi) \\ \tau_2^{OFF}(\beta, \varphi) &\in \arg \max_{\tau_2^{OFF}} W_{H2}^\circ[\tau_2^{OFF}, k(\beta, \varphi)] \\ \text{s.t.} \quad &W_{F2}^\circ[\tau_2(\beta, \varphi), k(\beta, \varphi)] + z_F^\circ(a; \varphi) \geq W_{F2}^\circ[\tau_2^{OFF}, k(\beta, \varphi)] + z_F^\circ(-a; \varphi), \end{aligned}$$

where we evaluate functions at “ β ” as a shorthand for evaluating them at their arguments under the solution to the problem at β .

Taking the limit as $\beta \rightarrow 0$ and $\varphi \rightarrow \hat{\varphi}$ and using Berge's theorem of the maximum implies

that, letting $\bar{x} \equiv \lim_{\beta \rightarrow 0, \varphi \rightarrow \hat{\varphi}} x(\beta, \varphi)$, the limits of solutions $\bar{\tau}_1$, $\bar{\tau}_2$, $\bar{\tau}_2^{OFF}$, and $\bar{k}$ satisfy

$$\begin{aligned} 0 &= W_{H1, \tau_1}^\circ(0, \hat{\varphi}) - \sum_{i \neq H} \bar{\lambda}_i^k \frac{dV_{i1, k}^\circ(0, \hat{\varphi})}{d\tau_1} \\ 0 &= W_{H1, k}^\circ(0, \hat{\varphi}) - \sum_{i \neq H} \bar{\lambda}_i^k \frac{dV_{i1, k}^\circ(0, \hat{\varphi})}{dk} \\ 0 &= W_{H2, \tau_2}^\circ(0, \hat{\varphi}) - \sum_{i \neq H} \bar{\lambda}_i^k \frac{dV_{i2, k}^\circ(0, \hat{\varphi})}{d\tau_2} \\ 0 &= W_{H2, \tau_2}^{\circ, OFF}(0, \hat{\varphi}) + \bar{\lambda}^{OFF} W_{F2, \tau_2}^{\circ, OFF}(0, \hat{\varphi}). \end{aligned}$$

If $\lambda_i^k = \bar{\lambda}_i^k$ and $\lambda^{OFF} = \bar{\lambda}^{OFF}$, then $\dot{\tau}_1$, $\dot{\tau}_2$, $\dot{\tau}_2^{OFF}$, and $\dot{k}$ solve this system, implying by our uniqueness assumption that $\dot{\tau}_1 = \bar{\tau}_1$, $\dot{\tau}_2 = \bar{\tau}_2$, $\dot{\tau}_2^{OFF} = \bar{\tau}_2^{OFF}$, and $\dot{k} = \bar{k}$. $\square$

H.2 Outline of the Computational Algorithm

This section describes the algorithm to solve the equilibrium levels of tariffs. The algorithm builds on Proposition 5 and solves the static optimization problems (H.3), (H.4), (H.5), and (H.2) to find the limiting optimal policy in the full model. The specific equilibrium solvers we use are described in Supplemental Appendix H.4. By Lerner symmetry, one trade instrument is redundant, so in all optimization problems we normalize one import tax to zero and optimize over the remaining instruments.

- i. We start by solving (H.3). In practice, we use the time-1 solver of Supplemental Appendix H.4.2, which takes tariffs as given. Capital is implicitly defined as a function of tariffs in our implementation, so the constraint in (H.3) holds by construction. We then numerically maximize Home welfare searching over import and export taxes. We denote the solution by $\dot{\tau}_1$ and store the implied equilibrium capital, $\dot{k}$.
- ii. We use the results from the previous step to find the Lagrange multipliers on the capital investment decision, $\dot{\lambda}_i^k$.
 - (a) Calculate the derivative of the marginal value of capital, $dV_{i1, k}^\circ/dk$, and the derivative of Home utility, dW_{H1}°/dk , evaluated at $(\dot{\tau}_1, \dot{k})$, numerically. To find these, we use the time-1 solver taking as given tariffs and capital defined in Supplemental Appendix H.4.3 and perturbing each element of $\dot{k}$ up and down by 1%.
 - (b) Calculate the Lagrange multipliers that are implicitly defined by

$$\sum_{i \neq H} \dot{\lambda}_i^k \cdot \frac{dV_{i1, k}^\circ}{dk} = W_{H1, k}^\circ,$$

by inverting the matrix of $dV_{i1,k}^\circ/dk$ using the non-Home rows.

- i. In practice, the derivatives $dV_{i1,k}^\circ/dk$ differ by orders of magnitude. Therefore, we rescale to ensure the matrix is computationally invertible. We then rescale back to find $\dot{\lambda}_i^k$.
- iii. We next set $\lambda_i^k = \dot{\lambda}_i^k$ from the previous step and solve (H.4) numerically, holding capital fixed at $\dot{k}$. We use the conflict solver in Supplemental Appendix H.4.5. We denote the solution by $\dot{\tau}_2$ and refer to the equilibrium by action a . We store Home consumption, $c_{H2}(a)$, and prices, $p_{H2}(a)$.
- iv. We define c^C as a map from consumption and prices into the positive reals.⁴ We search for the value of λ^{OFF} for which the solution to (H.5) results in a 0.2% drop in the value of c^C for Home compared to a , holding fixed capital, $\dot{k}$. That is, $c^C(c_{H2}^{OFF}, p_{H2}^{OFF}) = (1 - 0.002) c^C(c_{H2}(a), p_{H2}(a))$. We refer to this equilibrium by action $\neg a$. Our algorithm relies on c^C being monotone in λ^{OFF} , which we verify over the relevant range of λ^{OFF} .
- v. We then solve (H.2) numerically, again using the conflict solver in Supplemental Appendix H.4.5 and holding capital at $\dot{k}$. This is the time-2 deviation that maximizes Home's immediate utility. We denote the solution by $\dot{\tau}_2^*$ and use $*$ to refer to the equilibrium values.
- vi. We then compute how geopolitics affects time-1 policy. Rewriting (C.6) slightly, the optimal tariff at time 1 is given by

$$\begin{aligned}
& \tau_{1g} - \tau_{1g}^{ToT} \\
&= \kappa \left[\left(\tilde{r}_{F2}(\neg a) - m_{F2}(\neg a) \cdot \frac{\partial \tilde{p}_{F2}(\neg a)}{\partial k_F} - \tilde{r}_{F2}(a) + m_{F2}(a) \cdot \frac{\partial \tilde{p}_{F2}(a)}{\partial k_F} \right) \cdot \frac{\partial \check{k}_F}{\partial m_{H1g}} \right. \\
&\quad \left. + \left(m_{F2}(a) \cdot \frac{\partial \tilde{p}_{F2}(a)}{\partial m_{F2}} \cdot \frac{\partial \check{m}_{F2}(a)}{\partial k_H} - m_{F2}(\neg a) \cdot \frac{\partial \tilde{p}_{F2}(\neg a)}{\partial m_{F2}} \cdot \frac{\partial \check{m}_{F2}(\neg a)}{\partial k_H} \right) \cdot \frac{\partial \check{k}_{-H}}{\partial m_{H1g}} \right] / \check{p}_{1g}^w \\
&\quad + \left(\xi^\ell m_{H2}(a^\ell) \cdot \frac{\partial \check{p}_2^w(a^\ell)}{\partial k_{-H}} - \xi^* m_{H2}^* \cdot \frac{\partial \check{p}_2^{w*}}{\partial k_{-H}} \right) \cdot \frac{\partial \check{k}_{-H}}{\partial m_{H1g}} / \check{p}_{1g}^w,
\end{aligned} \tag{H.6}$$

⁴In practice, we define c^C as the consumption-for-consumption's-sake at time 2, which, recall, is given by $\prod_n (C_{Hn2}^C / \theta_{Hn}^C)^{\theta_{Hn}^C}$.

where

$$\xi^\ell \equiv \frac{\hat{\xi}}{\kappa_1} \times \frac{V_{H2,m}^\circ(a^\ell) \cdot m_2^\perp(a^\ell)}{\check{p}_2^w(a^\ell) \cdot m_2^\perp(a^\ell)}$$

$$\xi^* \equiv \frac{\hat{\xi}}{\kappa_1} \times \frac{V_{H2,m}^{*\circ} \cdot m_2^{*\perp}}{\check{p}_2^{w*} \cdot m_2^{*\perp}}.$$

We compute the dependence-building component up to κ and the credibility-building component up to $\hat{\xi}/\kappa_1$. We also compute the optimal industrial policy according to Theorem 2:

$$s_{1v} \propto \frac{\tilde{r}_{H2v}(m_{H2}(a^\ell), k_H) - \tilde{r}_{H2v}(m_{H2}^*, k_H)}{r_{H1v}}.$$

How we calculate $m_2^\perp(a^\ell)$, $m_2^{*\perp}$, and the derivatives is described in Supplemental Appendix H.3. In our calibration, the low state will be the off-path threat, $a^\ell = \neg a$, and so all commitment terms will compare the off-path equilibrium to the optimal deviation.

H.3 Tax Calculation Details

This section describes how we compute the components of (H.6).

H.3.1 Derivatives

All derivatives are computed by two-sided numerical differentiation, perturbing one element of the relevant vector up and down by 1% and re-solving the equilibrium. Cells that carry no trade, and capital varieties with a value of zero, are skipped and their derivatives set to zero.

Time-1 capital response. Using the peacetime solver that takes Home's trade vector as given (Supplemental Appendix H.4.4) at the trade vector implied by $\hat{\tau}_1$, we compute $\frac{\partial \check{k}_{-H}}{\partial m_{H1}}$.

Time-2 response to capital. We do not solve $\frac{\partial \check{m}_{F2}(\hat{a})}{\partial k_{-H}}$ and $\frac{\partial \check{p}_2^w(\hat{a})}{\partial k_{-H}}$ for all movements of capital as the dimensionality of capital is too large. Instead, we compute how capital moves in response to changes in Home's time-1 trade vector (Supplemental Appendix H.4.4) and then plug those changes in capital into the conflict solver that takes Home's trade vector as given (Supplemental Appendix H.4.6) at each of the three time-2 equilibria to compute $\frac{\partial \check{m}_{F2}(\hat{a})}{\partial k_{-H}} \cdot \frac{\partial \check{k}_{-H}}{\partial m_{H1}}$ and $\frac{\partial \check{p}_2^w(\hat{a})}{\partial k_{-H}} \cdot \frac{\partial \check{k}_{-H}}{\partial m_{H1}}$.

Foreign Meade derivatives. Using the conflict solver for a single country given its own trade quantities (Supplemental Appendix H.4.7), we compute the derivatives of Foreign's utility-denominated import prices with respect to Foreign's own capital, $\frac{\partial \check{p}_{F2}(\hat{a})}{\partial k_F}$, and its own trade vector, $\frac{\partial \check{p}_{F2}(\hat{a})}{\partial m_{F2}}$.

Home time-2 trade derivatives. At the off-path and deviation equilibria we also compute the response of all time-2 prices and quantities to Home’s own time-2 trade vector, $\frac{\partial \tilde{p}_2^w(\hat{a})}{\partial m_{H2}}$ and $\frac{\partial \tilde{m}_{F2}(\hat{a})}{\partial m_{H2}}$, using the conflict solver that takes Home’s trade vector as given (Supplemental Appendix H.4.6). These are used to pin down the multipliers ξ^ℓ and ξ^* .

H.3.2 Finding $m^\perp$ and ξ

As we are in the limit as $\beta \rightarrow 0$, time-2 trade has no effect on capital investment. Therefore, the only conditions from Appendix C.4.2 that $m_2^\perp(\hat{a})$ does not automatically satisfy are

$$\begin{aligned} 0 &= m_{H2}(\hat{a}) \cdot \frac{\partial \tilde{p}_2^w(\hat{a})}{\partial m_{H2}} \cdot m_2^\perp(\hat{a}) \\ 0 &= m_{F2}(\hat{a}) \cdot \frac{\partial \tilde{p}_{F2}(\hat{a})}{\partial m_{F2}} \cdot \frac{\partial \tilde{m}_{F2}(\hat{a})}{\partial m_{H2}} \cdot m_2^\perp(\hat{a}). \end{aligned}$$

We find a basis for the space of vectors that satisfy these conditions, $m_2^{b\perp}(\hat{a})$ for $b \in \{1, \dots, B\}$. For each basis vector, we calculate the implied $\underline{\xi}^b(\hat{a}) = (V_{H2,m}^\circ(\hat{a}) \cdot m_2^{b\perp}(\hat{a})) / (\tilde{p}_2^w(\hat{a}) \cdot m_2^{b\perp}(\hat{a}))$. This differs from ξ^ℓ as it is not scaled by $\hat{\xi}/\kappa_1$. $\underline{\xi}^b(\hat{a})$ varies slightly with b due to machine error, so we take $\underline{\xi}(\hat{a})$ to be the median of the $\underline{\xi}^b(\hat{a})$. We follow the same procedure to find $\underline{\xi}^*$.

H.4 Function Solvers

H.4.1 Preliminary: nesting within general framework

As an aside, we walk through the mapping between the parametric model presented in Section 6.1 and the general framework in Section 2—and, in particular, its extension to bystander countries in Appendix C.4—of which it is a special case.

The set of goods $\mathcal{G}$ includes, for each country i , a domestic composite for each sector n , and a differentiated variety for each sector n and destination j . The set of capital varieties $\mathcal{V}$ includes, for each sector n , one capital variety used in differentiated variety production, and $|\mathcal{I}|$ capital varieties used in composite production, one per origin country. Households’ consumption utility $u_{it}(\cdot)$ encompasses both their inelastic labor supply and their consumption preferences over domestic composite goods. The goods production possibilities frontier $G_{it}(\cdot, \cdot)$ describes the possible net output of goods—conditional on capital inputs—implied by the expressions for gross output of goods in (22) and (24). The capital production possibilities frontier $\Lambda_{i1}(\cdot, \cdot)$ describes the possible capital output—conditional on investment—stated in (23) and (25).

To fully align the quantitative analysis with our general framework, we study a superficially altered version of the quantitative model presented in the main text. First, we assume that

production firms rent capital on competitive markets rather than owning it themselves. Second, we assume that it is not relationship capital K_{ijn} that directly enters the production function for sectoral composites but rather normalized relationship capital K_{ijn}/K_{jn} , where K_{jn} is a homothetic aggregator of $\{K_{ijn}\}_{i \in \mathcal{I}}$. This ensures the problem of sectoral composite producers has a well-defined solution when they rent capital rather than owning it directly. However, it has no other consequences since our assumption on the formation of relationship capital ensures K_{jn} is always equal to one in equilibrium. The equilibrium solver of Supplemental Appendix H.4.5 checks this restriction numerically at every candidate policy and stops if it is violated.

In what follows, we adopt the following notation for prices (notation for quantities was introduced in Section 6). We let w_i denote wages, r_{in} denote production capital prices, r_{ijn} denote relationship capital prices, p_{ijn} denote origin-sector variety prices in the destination j , P_{in} denote sectoral composite prices, τ_{ijn}^m denote ad valorem import tariffs, and τ_{ijn}^x denote ad valorem export subsidies. We assume there are no domestic taxes, i.e., $\tau_{iin}^m = \tau_{iin}^x = 0$.

H.4.2 Numerical algorithm for equilibrium in peacetime static model

In the static model that represents the peacetime period, capital can adjust freely, subject to the investment technology. We now describe the series of equilibrium conditions that determine all endogenous variables, including capital, given trade taxes.

Sectoral composite production with endogenous relationship capital. As a preliminary step, we characterize the level of relationship capital that emerges in the equilibrium of the static model. We substitute this level of capital into the production function for sectoral composites, arriving at a new production function that already incorporates equilibrium outcomes in the relationship capital market.

The production function for relationship capital in the main text implies that for some $\{\kappa_{in}\}$, we have

$$r_{oin} = \kappa_{in} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\sigma-1)/(\bar{\sigma}-\sigma)}$$

for all (o, i, n) triplets with $\theta_{oin}^K > 0$. Combining this rental rate with the production function of sectoral composites implies that relationship capital demands satisfy, for some $\{\tilde{\kappa}_{in}\}$,

$$K_{oin} = \tilde{\kappa}_{in} (\theta_{oin}^K)^{\sigma/\bar{\sigma}} (x_{oin})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}$$

for all (o, i, n) triplets with $\theta_{oin}^K > 0$, where x_{oin} is the quantity of origin- o destination- i sector- n goods the sector- n composite producer in i uses as an input.

Letting $K_{in} \equiv \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$, this implies

$$\frac{K_{oin}}{K_{in}} = \frac{(\theta_{oin}^K)^{\sigma/\bar{\sigma}} (x_{oin})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o' \in \mathcal{I}: \theta_{o'in}^K > 0} (\theta_{o'in}^K)^{1/\bar{\sigma}} (x_{o'in})^{(\bar{\sigma}-1)/\bar{\sigma}} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}}$$

for all (o, i, n) triplets with $\theta_{oin}^K > 0$. Combining this observation with the capital production constraint,

$$\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} = 1,$$

implies $K_{in} = 1$, so that

$$K_{oin} = \frac{(\theta_{oin}^K)^{\sigma/\bar{\sigma}} (x_{oin})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o' \in \mathcal{I}: \theta_{o'in}^K > 0} (\theta_{o'in}^K)^{1/\bar{\sigma}} (x_{o'in})^{(\bar{\sigma}-1)/\bar{\sigma}} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}}. \quad (\text{H.7})$$

Finally, we substitute back into the production function for sectoral composites. This implies the following after-relationship-capital-optimization production function for sectoral composites

$$Q_{in} = \left[\sum_{o \in \mathcal{I}} (\theta_{oin}^K)^{1/\bar{\sigma}} (x_{oin})^{(\bar{\sigma}-1)/\bar{\sigma}} \right]^{\bar{\sigma}/(\bar{\sigma}-1)}. \quad (\text{H.8})$$

Goods prices. We now characterize the prices of all goods in terms of wages w_i and rental rates r_{in} of production capital in each country i and sector n . The origin-destination-sector production function in the main text and the after-relationship-capital-optimization production function for sectoral composites derived above imply

$$p_{ijn} = \frac{1 + \tau_{ijn}^m}{1 + \tau_{ijn}^x} (\phi_{in}^q)^{-1} (w_i)^{\theta_{in}^\ell} (r_{in})^{\theta_{in}^k} \prod_{n' \in \mathcal{N}} (P_{in'})^{\theta_{in'n}^Q} \quad (\text{H.9})$$

$$P_{in} = \left[\sum_{o \in \mathcal{I}} \theta_{oin}^K (p_{oin})^{1-\bar{\sigma}} \right]^{1/(1-\bar{\sigma})}.$$

Spending. The expenditures of the country- i household on consumption of sectoral composites, C_i , are equal to household labor income, plus profits, plus trade tax revenues T_i in i :

$$C_i = w_i \ell_i + \sum_{n \in \mathcal{N}} \theta_{in}^\pi r_{in} k_{in} + T_i, \quad (\text{H.10})$$

where $\theta_{in}^\pi \equiv 1 - \sum_{n' \in \mathcal{N}} \theta_{in'n}^\ell$. The utility function in the main text implies that household expenditure in country i on sectoral composite n is given by

$$C_{in} = \theta_{in}^C C_i. \quad (\text{H.11})$$

Note that capital rental expenditures $r_{in}k_{in}$ can be represented in terms of the revenues of country- i firms on sales of product n to country j , Y_{ijn} , due to the structure of origin-destination-sector good production,

$$R_{in} = r_{in}k_{in} = \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}. \quad (\text{H.12})$$

We denote aggregate expenditures on country i 's sector- n composite by X_{in} . These expenditures include both domestic consumption expenditures and the intermediate input expenditures of domestic firms, both goods and capital producers. The structure of goods and capital production implies

$$X_{in} = C_{in} + \sum_{n' \in \mathcal{N}} \left(\theta_{inn'}^Q + \theta_{in'}^k \theta_{inn'}^\ell \right) \sum_{j \in \mathcal{I}} Y_{ijn'}. \quad (\text{H.13})$$

Note these revenues are equal to expenditures X_{ijn} on i 's exports of n in j , net of trade taxes:

$$Y_{ijn} = \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn}. \quad (\text{H.14})$$

Next we observe that the origin-destination-sector expenditures X_{ijn} can be written in terms of sectoral composite expenditures, using the structure of the after-relationship-capital-optimization production function described in (H.8):

$$X_{ijn} = \zeta_{ijn} X_{jn} \quad (\text{H.15})$$

$$\text{where } \zeta_{ijn} = \frac{\theta_{ijn}^K (p_{ijn})^{1-\bar{\sigma}}}{(P_{jn})^{1-\bar{\sigma}}}.$$

This expression also allows us to express trade tax revenues in terms of sectoral composite expenditures:

$$\begin{aligned} T_i &= \sum_{o \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} X_{oin} - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn} \\ &= \sum_{o \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} \zeta_{oin} X_{in} - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn} X_{jn}. \end{aligned} \quad (\text{H.16})$$

Combining (H.10)-(H.16) implies a linear system for sectoral composite spending X_{in} in terms of prices and exogenous variables:

$$X = [\text{id} - \mathcal{X}]^{-1} X_0 \quad (\text{H.17})$$

where $[X]_{in} = X_{in}$

$$[X_0]_{in} = \theta_{in}^C w_i \ell_i$$

$$[\mathcal{X}]_{in,jn'} = \theta_{in}^C \mathbb{1}_{i=j} \sum_{o \in \mathcal{I}} \frac{\tau_{oin'}^m}{1 + \tau_{oin'}^m} \zeta_{oin'} + \left(\left(\theta_{inn'}^Q + (\theta_{inn'}^\iota + \theta_{in}^C \theta_{in'}^\pi) \theta_{in'}^k \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} - \theta_{in}^C \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \right) \zeta_{ijn'}.$$

Factor markets. Finally, we characterize factor demand clearing in terms of aggregate spending on sectoral composites. The production function for origin-destination-sector goods, together with (H.14) and (H.15), implies aggregate labor spending of

$$W_i = \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}. \quad (\text{H.18})$$

Factor market clearing implies

$$w_i \ell_i = W_i. \quad (\text{H.19})$$

The production function for capital implies capital supply satisfies

$$r_{in} k_{in} = r_{in} \left(\phi_{in}^k \prod_{n' \in \mathcal{N}} (r_{in}/P_{in'})^{\theta_{in'n}^\iota} \right)^{1/\theta_{in}^\pi}. \quad (\text{H.20})$$

Equivalently, $r_{in} = (\phi_{in}^k)^{-1} \prod_{n'} (P_{in'})^{\theta_{in'n}^\iota} (R_{in})^{\theta_{in}^\pi}$, which is the form used in the iteration below.

Numerical algorithm. We iterate over wages, production capital rental rates, and sectoral composite prices in outer and inner loops indexed by a and b , respectively. We let $\{w_i^a\}$ and $\{r_{in}^a\}$ denote wages and capital rental rates, respectively, in the a^{th} outer loop and let $\{P_{in}^{a,b}\}$ denote sectoral composite prices in the a^{th} outer loop and b^{th} inner loop. We proceed as follows.⁵

⁵Two conventions are common to all six solvers. First, prohibitive trade taxes are admitted and are represented by $\tau_{ijn}^m = \infty$ for an import embargo and $\tau_{ijn}^x = -1$ for an export embargo. On such cells the variety price p_{ijn} is infinite and the trade flow is zero; in the trade tax revenue expression the share $\tau_{ijn}^m/(1 + \tau_{ijn}^m)$ is set to one. Infinite prices are permitted only on these cells, and the solvers stop if any other output is not a finite real number. Second, every solver accepts initial guesses for the factor prices and, where

- i. At $a = 1$, initialize all $w_i^{a=1}$ and $r_{in}^{a=1}$ at their initial guesses, or at one if none is supplied.
- ii. Solve for sectoral composite prices given wages as follows.

(a) At $b = 1$, initialize all $P_{in}^{a,b=1} = 1$ if $a = 1$ or $P_{in}^{a,b=1} = P_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

(b) Define $\tilde{P}_{in}^{a,b}$ using (H.9), setting

$$p_{ijn}^{a,b} = \frac{1 + \tau_{ijn}^m (\phi_{in}^q)^{-1} (w_i^a)^{\theta_{in}^\ell} (r_{in}^a)^{\theta_{in}^k}}{1 + \tau_{ijn}^x} \prod_{n' \in \mathcal{N}} (P_{in'}^{a,b})^{\theta_{in'n}^Q}$$

$$\tilde{P}_{in}^{a,b} = \left[\sum_{o \in \mathcal{I}} \theta_{oin}^K (p_{oin}^{a,b})^{1-\bar{\sigma}} \right]^{1/(1-\bar{\sigma})}.$$

(c) For some numerical parameter $\text{tol}_{\text{inner}}$, test whether

$$\max_{i \in \mathcal{I}, n \in \mathcal{N}} \left| \log \tilde{P}_{in}^{a,b} - \log P_{in}^{a,b} \right| < \text{tol}_{\text{inner}}.$$

- i. If not, update prices according to

$$P_{in}^{a,b+1} = \exp \left[\log P_{in}^{a,b} + \text{step}_{\text{inner}} \left(\log \tilde{P}_{in}^{a,b} - \log P_{in}^{a,b} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner}} > 0$. Increase b by one and return to step 2.b.

- ii. If so, set $P_{in}^a \equiv P_{in}^{a,b}$, $p_{ijn}^a \equiv p_{ijn}^{a,b}$, and proceed to step 3.

- iii. Compute sectoral composite spending X_{in}^a according to (H.15) and (H.17), setting

$$X^a = [\text{id} - \mathcal{X}^a]^{-1} X_0^a$$

where $[X^a]_{in} = X_{in}^a$

$$[X_0^a]_{in} = \theta_{in}^C w_i^a \ell_i$$

$$[\mathcal{X}^a]_{in,jn'} = \mathbb{1}_{i=j} \theta_{in}^C \left(\sum_{o \in \mathcal{I}} \frac{\tau_{ojn'}^m}{1 + \tau_{ojn'}^m} \zeta_{ojn'}^a \right)$$

$$+ \left(\theta_{inn'}^Q + (\theta_{inn'}^\ell + \theta_{in}^C \theta_{in'}^\pi) \theta_{in'}^k \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a - \theta_{in}^C \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a$$

$$\zeta_{ijn}^a = \frac{\theta_{ijn}^K (p_{ijn}^a)^{1-\bar{\sigma}}}{(P_{jn}^a)^{1-\bar{\sigma}}}.$$

relevant, the Lagrange multipliers; when a solver is called repeatedly along an optimization path we start it from the previously converged values.

iv. Define $\tilde{w}_i^a$ and $\tilde{r}_{in}^a$ using (H.18), (H.19), (H.12), and (H.20), setting

$$\begin{aligned}\tilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \tilde{r}_{in}^a &= (\phi_{in}^k)^{-1} \prod_{n' \in \mathcal{N}} (P_{in'}^a)^{\theta_{in'}^k} \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right)^{\theta_{in}^\pi} \\ \text{where } Y_{ijn}^a &= \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a.\end{aligned}$$

v. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\begin{aligned}\max_{i \in \mathcal{I}} |\log \tilde{w}_i^a - \log w_i^a| &< \text{tol}_{\text{outer}} \\ \text{and } \max_{i \in \mathcal{I}, n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| &< \text{tol}_{\text{outer}}.\end{aligned}$$

(a) If not, update wages and capital rental rates according to

$$\begin{aligned}w_i^{a+1} &= \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{w}_i^a \\ r_{in}^{a+1} &= \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{r}_{in}^a \\ \text{where } \hat{w}_i^a &= \exp [\log w_i^a + \text{step}_{\text{outer}} (\log \tilde{w}_i^a - \log w_i^a)] \\ \hat{r}_{in}^a &= \exp [\log r_{in}^a + \text{step}_{\text{outer}} (\log \tilde{r}_{in}^a - \log r_{in}^a)] \\ k_{in}^a &= \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right) / r_{in}^a\end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2. The constant ξ is the calibrated level of world factor income and fixes the numeraire.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{jn}^a$, $k_{in} = k_{in}^a$, and, using (H.7), (H.13), (H.14), and the production function for

production capital in the main text, set

$$\begin{aligned}
K_{ijn} &= \begin{cases} \frac{(\theta_{ijn}^K)^{\sigma/\bar{\sigma}} (X_{ijn}/p_{ijn})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o \in \mathcal{I}: \theta_{ojn}^K > 0} (\theta_{ojn}^K)^{1/\bar{\sigma}} (X_{ojn}/p_{ojn})^{(\bar{\sigma}-1)/\bar{\sigma}} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}} & \text{if } \theta_{ijn}^K > 0 \\ 0 & \text{if } \theta_{ijn}^K = 0 \end{cases} \\
C_i &= \sum_{n \in \mathcal{N}} X_{in} - \sum_{n, n' \in \mathcal{N}} \left(\theta_{inn'}^Q + \theta_{in'}^k \theta_{inn'}^\ell \right) \sum_{j \in \mathcal{I}} Y_{ijn'}^a \\
\iota_{in'n} &= \theta_{in'n}^\ell r_{in} k_{in} / P_{in'}.
\end{aligned}$$

Note that setting $K_{ijn} = 0$ when $\theta_{ijn}^K = 0$ is without loss of generality as K_{ijn} never affects production in such cases. The solver stops with an error if any $K_{ijn} \neq 0$ is allocated to an origin from which there are no imports.

H.4.3 Numerical algorithm for equilibrium in peacetime static model given capital

Next we describe the peacetime static model, taking capital and tariffs as given and computing the necessary investment to make that capital. Much remains unchanged from Supplemental Appendix H.4.2. Thus, in this section we simply describe what has changed.

Goods prices. We begin by characterizing the prices of all goods in terms of the factor prices $\{w_i\}$ and $\{r_{in}\}$ and relationship capital $\{K_{ijn}\}$. The price of the varieties follows (H.9), but the price of the composite now depends on the fixed capital,

$$P_{in} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (K_{oin}/K_{in}) (p_{oin})^{1-\sigma} \right]^{1/(1-\sigma)}, \quad (\text{H.21})$$

where $K_{in} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$.

Spending. Firm profits are no longer a fixed share of capital revenue. Thus, country- i expenditures are not (H.10), but

$$C_i = w_i \ell_i + \sum_{n \in \mathcal{N}} \pi_{in} + T_i, \quad (\text{H.22})$$

where π_{in} is the profit of the capital producer in sector n . The investment firm needs to invest enough to create k_{in} capital. Defining

$$\lambda_{in} \equiv (\phi_{in}^k)^{-\frac{1}{\sum_{n'} \theta_{in'n}^\ell}} (k_{in})^{\frac{1}{\sum_{n'} \theta_{in'n}^\ell}} \left(\prod_{n'} (P_{in'})^{\theta_{in'n}^\ell} \right)^{\frac{1}{\sum_{n'} \theta_{in'n}^\ell}}, \quad (\text{H.23})$$

spending on investment goods can be written $P_{in'} \ell_{in'n} = \theta_{in'n}^\ell \lambda_{in}$. Profits are then $\pi_{in} = \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn} - \sum_{n'} \theta_{in'n}^\ell \lambda_{in}$. Household spending on sectors remains unchanged, (H.11).

Spending on country i sector n is then given by

$$X_{in} = C_{in} + \sum_{n' \in \mathcal{N}} \theta_{inn'}^Q \sum_{j \in \mathcal{I}} Y_{ijn'} + \sum_{n'} \theta_{inn'}^\ell \lambda_{in'}, \quad (\text{H.24})$$

rather than (H.13).

Next we observe that the origin-destination-sector expenditures X_{ijn} are no longer (H.15), but

$$X_{ijn} = \zeta_{ijn} X_{jn} \quad (\text{H.25})$$

where $\zeta_{ijn} = \frac{(K_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(P_{jn})^{1-\sigma}}.$

Tax revenue is still given by (H.16). Combining these adjusted spending equations implies a linear system for sectoral composite spending X_{in} in terms of prices and exogenous variables:

$$X = [\text{id} - \mathcal{X}]^{-1} X_0 \quad (\text{H.26})$$

where $[X]_{in} = X_{in}$

$$[X_0]_{in} = \theta_{in}^C w_i \ell_i + \sum_{n'} \theta_{inn'}^\ell \lambda_{in'} - \theta_{in}^C \sum_{n''} \sum_{n'} \theta_{in''n'}^\ell \lambda_{in'}$$

$$[\mathcal{X}]_{in,jn'} = \theta_{in}^C \mathbb{1}_{i=j} \sum_{o \in \mathcal{I}} \frac{\tau_{oin'}^m}{1 + \tau_{oin'}^m} \zeta_{oin'}$$

$$+ \left(\left(\theta_{in}^C \theta_{in'}^k + \theta_{inn'}^Q \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} - \theta_{in}^C \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \right) \zeta_{ijn'}.$$

Factor markets. Labor market clearing is unchanged, so (H.18) and (H.19) still hold. Capital supply is now fixed, so (H.12) is now a market clearing condition.

Budget Lagrange multiplier. The Lagrange multiplier on the budget constraint in peacetime is given by

$$\psi_i = \frac{(C_i)^{-1/\rho}}{\left(\prod_n (P_{in})^{\theta_{in}^C}\right)^{(\rho-1)/\rho}}. \quad (\text{H.27})$$

Numerical algorithm. We iterate over wages, production capital rental rates, and sectoral composite prices in outer and inner loops indexed by a and b , respectively. We proceed as follows.

- i. At $a = 1$, initialize all $w_i^{a=1}$ and $r_{in}^{a=1}$ at their supplied initial guesses.
- ii. Solve for sectoral composite prices given wages exactly as in Supplemental Appendix [H.4.2](#), except that $\tilde{P}_{in}^{a,b}$ is defined using [\(H.21\)](#), i.e., with weights K_{oin}/K_{in} and elasticity σ in place of θ_{oin}^K and $\bar{\sigma}$.
- iii. Compute sectoral composite spending X_{in}^a according to [\(H.25\)](#) and [\(H.26\)](#), setting

$$\begin{aligned} X^a &= [\text{id} - \mathcal{X}^a]^{-1} X_0^a \\ [X_0^a]_{in} &= \theta_{in}^C w_i^a \ell_i + \sum_{n'} \theta_{inn'}^\ell \lambda_{in'}^a - \theta_{in}^C \sum_{n'} \left(\sum_{n''} \theta_{in''n'}^\ell \right) \lambda_{in'}^a \\ [\mathcal{X}^a]_{in,jn'} &= \mathbb{1}_{i=j} \theta_{in}^C \left(\sum_{o \in \mathcal{I}} \frac{\tau_{ojn'}^m}{1 + \tau_{ojn'}^m} \zeta_{ojn'}^a \right) \\ &\quad + \left(\theta_{in}^C \theta_{in'}^k + \theta_{inn'}^Q \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a - \theta_{in}^C \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a \\ \zeta_{ijn}^a &= \frac{(K_{ijn}/K_{jn}) (p_{ijn}^a)^{1-\sigma}}{(P_{jn}^a)^{1-\sigma}} \\ \lambda_{in'}^a &= (\phi_{in'}^k)^{-\frac{1}{\sum_n \theta_{inn'}^\ell}} (k_{in'})^{\frac{1}{\sum_n \theta_{inn'}^\ell}} \left(\prod_n (P_{in}^a)^{\theta_{inn'}^\ell} \right)^{\frac{1}{\sum_n \theta_{inn'}^\ell}}. \end{aligned}$$

- iv. Define $\tilde{w}_i^a$ and $\tilde{r}_{in}^a$ using [\(H.18\)](#), [\(H.19\)](#), and [\(H.12\)](#), setting

$$\begin{aligned} \tilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \tilde{r}_{in}^a &= \frac{1}{k_{in}} \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \text{where } Y_{ijn}^a &= \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a. \end{aligned}$$

v. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\max_{i \in \mathcal{I}} |\log \tilde{w}_i^a - \log w_i^a| < \text{tol}_{\text{outer}}$$

and $\max_{i \in \mathcal{I}, n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| < \text{tol}_{\text{outer}}.$

(a) If not, update wages and rental rates according to

$$w_i^{a+1} = \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{w}_i^a$$

$$r_{in}^{a+1} = \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{r}_{in}^a$$

where $\hat{w}_i^a = \exp [\log w_i^a + \text{step}_{\text{outer}} (\log \tilde{w}_i^a - \log w_i^a)]$

$\hat{r}_{in}^a = \exp [\log r_{in}^a + \text{step}_{\text{outer}} (\log \tilde{r}_{in}^a - \log r_{in}^a)]$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{jn}^a$, and set

$$C_i = \sum_{n \in \mathcal{N}} X_{in} - \sum_{n, n' \in \mathcal{N}} \theta_{inn'}^Q \sum_{j \in \mathcal{I}} Y_{ijn'}^a - \sum_n \left(\sum_{n'} \theta_{in'n}^\iota \right) \lambda_{in}^a$$

$$\iota_{in'n} = \theta_{in'n}^\iota \lambda_{in}^a / P_{in'}.$$

(c) Finally, we also return the net rental rate of capital. For relationship capital, this is

$$r_{ijn} = \frac{X_{jn}}{K_{ijn}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{ijn}^K > 0} \left(\frac{(K_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(P_{jn})^{1-\sigma}} - \frac{(\theta_{ijn}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right).$$

For production capital, this is the difference between the rental rate and the cost of adding another unit,

$$\bar{r}_{in} = r_{in} - \frac{\lambda_{in}^a}{k_{in}}.$$

H.4.4 Numerical algorithm for equilibrium in peacetime static model given Home trade

In the static model that represents the peacetime period, capital can adjust freely, subject to the investment technology. We now describe the series of equilibrium conditions that determine all endogenous variables, including capital, given Home's trade vectors, m_{Hjn} and

m_{iHn} . The characterization of relationship capital is identical to the one in Supplemental Appendix H.4.2, so (H.7) and (H.8) continue to hold.

Goods prices. For $i \neq H$, (H.9) remains unchanged. For H , prices are determined instead by market clearing.

Spending. Country- $i \neq H$'s expenditures now include an additional transfer,

$$C_i = w_i \ell_i + \sum_{n \in \mathcal{N}} \theta_{in}^\pi r_{in} k_{in} + T_i + \Gamma_i \quad (\text{H.28})$$

where $\Gamma_i = \frac{1}{|\mathcal{I}|-1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}}{1 + \tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}}{1 + \tau_{oHn}^m} m_{oHn} \right]$.

The term Γ_i represents the transfer from Home to country i to fund its trade deficit or surplus. Equations (H.11)-(H.16) still hold for $i \neq H$, and (H.15) applies only to flows with $i \neq H$ and $j \neq H$. Flows including H are given by

$$X_{ijn} = p_{ijn} m_{ijn} \quad \text{for } i = H \text{ or } j = H. \quad (\text{H.29})$$

Combining these equations implies a linear system for the sectoral composite spending of the non-Home countries:

$$X_{-H} = [\text{id} - \mathcal{X}_{-H}]^{-1} X_{-H0} \quad (\text{H.30})$$

where $[X_{-H}]_{in} = X_{in}$

$$\begin{aligned} [X_{-H0}]_{in} &= \theta_{in}^C (w_i \ell_i + \Gamma_i) + \theta_{in}^C \sum_{n' \in \mathcal{N}} \frac{\tau_{Hin'}^m}{1 + \tau_{Hin'}^m} p_{Hin'} m_{Hin'} \\ &\quad + \sum_{n'} \left(-\theta_{in}^C \frac{\tau_{iHn'}^x}{1 + \tau_{iHn'}^m} + \left(\theta_{inn'}^Q + (\theta_{inn'}^\ell + \theta_{in}^C \theta_{in'}^\pi) \theta_{in'}^k \right) \frac{1 + \tau_{iHn'}^x}{1 + \tau_{iHn'}^m} \right) p_{iHn'} m_{iHn'} \\ [\mathcal{X}_{-H}]_{in,jn'} &= \theta_{in}^C \mathbf{1}_{i=j} \sum_{o \neq H} \frac{\tau_{oin'}^m}{1 + \tau_{oin'}^m} \zeta_{oin'} \\ &\quad + \left(\left(\theta_{inn'}^Q + (\theta_{inn'}^\ell + \theta_{in}^C \theta_{in'}^\pi) \theta_{in'}^k \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} - \theta_{in}^C \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \right) \zeta_{ijn'}. \end{aligned}$$

Factor markets. Factor markets remain unchanged, so (H.18), (H.19), and (H.20) all hold for $i \neq H$.

Numerical algorithm. We iterate over wages, production capital rental rates, Home export prices, and sectoral composite prices in outer and inner loops indexed by a and b , respectively. We proceed as follows.

- i. At $a = 1$, initialize all $w_i^{a=1}$, $r_{in}^{a=1}$, and $p_{Hjn}^{a=1}$ at their supplied initial guesses.
- ii. Solve for sectoral composite prices given factor prices as in Supplemental Appendix H.4.2, with two changes: the Home row of variety prices is fixed at $p_{Hjn}^{a,b} = p_{Hjn}^a$ rather than computed from (H.9), and the Home row of composite prices is set to $\tilde{P}_{Hn}^{a,b} = 1$, since Home's own price index is not used.
- iii. Compute sectoral composite spending X_{in}^a for $i \neq H$ according to (H.15) and (H.30), evaluating ζ_{ijn}^a , p_{ijn}^a and P_{jn}^a at the prices from step 2 and setting

$$\Gamma_i^a = \frac{1}{|\mathcal{I}|-1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}^a}{1 + \tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}^a}{1 + \tau_{oHn}^m} m_{oHn} \right].$$

- iv. Define $\tilde{w}_i^a$, $\tilde{r}_{in}^a$, and $\tilde{p}_{Hin}^a$ for $i \neq H$ using (H.18), (H.19), and (H.20), setting

$$\begin{aligned} \tilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \tilde{r}_{in}^a &= (\phi_{in}^k)^{-1} \prod_{n' \in \mathcal{N}} (P_{in'}^a)^{\theta_{in'}^t} \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right)^{\theta_{in}^\pi} \\ \tilde{p}_{Hin}^a &= \frac{1}{m_{Hin}} \zeta_{Hin}^a X_{in}^a \\ \text{where } Y_{ijn}^a &= \begin{cases} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a & \text{for } j \neq H \\ \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} p_{iHn}^a m_{iHn} & \text{for } j = H \end{cases}. \end{aligned}$$

Without loss of generality, set $\tilde{w}_H^a = \tilde{r}_{Hn}^a = \tilde{p}_{HHn}^a = 1$.

- v. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\begin{aligned} \max_{i \neq H} |\log \tilde{w}_i^a - \log w_i^a| &< \text{tol}_{\text{outer}} \\ \max_{i \neq H, n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| &< \text{tol}_{\text{outer}} \\ \text{and } \max_{i \neq H, n \in \mathcal{N}} |\log \tilde{p}_{Hin}^a - \log p_{Hin}^a| &< \text{tol}_{\text{outer}}. \end{aligned}$$

(a) If not, update according to

$$\begin{aligned} w_i^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{w}_i^a \\ r_{in}^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{r}_{in}^a \\ p_{Hin}^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{p}_{Hin}^a, \end{aligned}$$

where $\hat{w}$, $\hat{r}$, and $\hat{p}$ are the log-linear updates of step $\text{step}_{\text{outer}}$ defined as in Supplemental Appendix H.4.2 and $k_{in}^a = \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right) / r_{in}^a$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{jn}^a$ for flows not involving Home and $X_{ijn} = p_{ijn} m_{ijn}$ for flows that do, $k_{in} = k_{in}^a$, and recover K_{ijn} , C_i , and $\iota_{in'n}$ exactly as in step 5.b of Supplemental Appendix H.4.2. None of the values for $i = H$, nor K_{ijn} for $j = H$, are used.

H.4.5 Numerical algorithm for equilibrium in conflict static model

In the static model that represents the conflict period, capital is exogenous. We now describe the series of equilibrium conditions that determine all endogenous variables, given capital and trade taxes (which are also exogenous). As a preliminary check, the solver verifies that the relationship capital aggregates satisfy $K_{jn} = 1$ at the capital stocks it is handed, and stops if they do not, since this is the restriction that makes the sectoral composite problem well defined.

Goods prices. We begin by characterizing the prices of all goods in terms of the factor prices $\{w_i\}$ and $\{r_{in}\}$ and relationship capital $\{K_{ijn}\}$.

$$\begin{aligned} p_{ijn} &= \frac{1 + \tau_{ijn}^m}{1 + \tau_{ijn}^x} (\phi_{in}^q)^{-1} (w_i)^{\theta_{in}^q} (r_{in})^{\theta_{in}^k} \prod_{n' \in \mathcal{N}} (P_{in'})^{\theta_{in'n}^Q} \\ P_{in} &= \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (K_{oin}/K_{in}) (p_{oin})^{1-\sigma} \right]^{1/(1-\sigma)}, \end{aligned} \tag{H.31}$$

where $K_{in} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$.

Consumer demand. We next characterize country- i 's household expenditures on sectoral composites, C_{in} , in terms of prices and the Lagrange multiplier ψ_i on the household budget

constraint. The two-part utility function presented in the main text implies that the households' consumption expenditures satisfy

$$C_{in} = \theta_{in}^C P_i^c \tilde{c}_i + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^\iota}{1 - \theta_{in'}^\pi} P_{in'}^\iota \tilde{c}_{in'} \quad (\text{H.32})$$

where $\tilde{c}_i = (\psi_i P_i^c)^{-\rho}$

$$\tilde{c}_{in} = (\psi_i P_{in}^\iota / \tilde{\nu}_{in})^{-1/\theta_{in}^\pi}$$

$$P_i^c = \prod_{n \in \mathcal{N}} (P_{in})^{\theta_{in}^C}$$

$$P_{in}^\iota = \prod_{n' \in \mathcal{N}} (P_{in'})^{\theta_{in'n}^\iota / (1 - \theta_{in}^\pi)}$$

$$\tilde{\nu}_{in} \equiv \nu_{in} \phi_{in}^k (1 - \theta_{in}^\pi)^{\theta_{in}^\pi}.$$

Spending. We next use the above description of consumer demand to characterize total expenditures on each sectoral composite good in terms of prices and budget constraint Lagrange multipliers. Letting Y_{ijn} denote the revenues of country- i firms on sales in sector n to country j , the production function in the main text implies

$$X_{in} = C_{in} + \sum_{j \in \mathcal{I}} \sum_{n' \in \mathcal{N}} \theta_{inn'}^Q Y_{ijn'}, \quad (\text{H.33})$$

and these revenues are equal to expenditures net of trade taxes,

$$Y_{ijn} = \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn}. \quad (\text{H.34})$$

Next we observe that the origin-destination-sector expenditures X_{ijn} can be written in terms of expenditures on sectoral composites,

$$X_{ijn} = \zeta_{ijn} X_{jn} \quad (\text{H.35})$$

where $\zeta_{ijn} = \frac{(K_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(P_{jn})^{1-\sigma}}.$

Combining (H.33)-(H.35) implies a linear system for sectoral composite spending X_{in} in terms of prices and exogenous variables.

$$X = [\text{id} - \mathcal{X}]^{-1} C \quad (\text{H.36})$$

$$\text{where } [X]_{in} = X_{in}$$

$$[C]_{in} = C_{in}$$

$$[\mathcal{X}]_{in,jn'} = \theta_{inn'}^Q \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}.$$

Lagrange multipliers on household budget constraints. We next show how the Lagrange multipliers on household budget constraints are determined by consumption, aggregate spending, and factor prices. Each country i has tax revenues

$$T_i = \sum_{o \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} X_{oin} - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn}, \quad (\text{H.37})$$

where, on cells with a prohibitive import tariff, the share $\tau_{oin}^m/(1 + \tau_{oin}^m)$ is understood to equal one. The Lagrange multiplier on each household budget constraint must be such that, given these revenues, the budget constraint holds:

$$\sum_{n \in \mathcal{N}} C_{in} = w_i \ell_i + \sum_{n \in \mathcal{N}} r_{in} k_{in} + T_i. \quad (\text{H.38})$$

Factor markets. Finally, we characterize factor demand and factor market clearing in terms of prices and aggregate spending on sectoral composites.

$$W_i = \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn} \quad (\text{H.39})$$

$$R_{in} = \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}$$

$$w_i \ell_i = W_i, \quad r_{in} k_{in} = R_{in}.$$

Any given variety of relationship capital can, in any country, only be allocated to a single sectoral composite producer. However, it will nonetheless be useful to compute the implicit rental rate r_{ijn} associated with each such variety K_{ijn} . The production functions for sectoral

composites imply

$$r_{ijn} = \frac{X_{jn}}{K_{ijn}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{ijn}^K > 0} \left(\frac{(K_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(P_{jn})^{1-\sigma}} - \frac{(\theta_{ijn}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right). \quad (\text{H.40})$$

Note this can, in general, be negative, as relationship capital with one origin detracts from relationship capital with another origin.

Numerical algorithm. We iterate over wages, production capital rental rates, sectoral composite prices, and Lagrange multipliers on household budget constraints in one outer and two successive inner loops indexed by a , b , and b' , respectively. We proceed as follows.

- i. At $a = 1$, initialize all $w_i^{a=1}$ and $r_{in}^{a=1}$ at their supplied initial guesses.
- ii. Solve for sectoral composite prices given factor prices as in Supplemental Appendix [H.4.2](#), with $\tilde{P}_{in}^{a,b}$ defined using [\(H.31\)](#) and tolerance $\text{tol}_{\text{inner},1}$ and step $\text{step}_{\text{inner},1}$.
- iii. Compute true-consumption and investment-as-consumption price indices P_i^{ca} and $P_{in}^{\iota a}$ according to [\(H.32\)](#), setting

$$P_i^{ca} = \prod_{n \in \mathcal{N}} (P_{in}^a)^{\theta_{in}^C}, \quad P_{in}^{\iota a} = \prod_{n' \in \mathcal{N}} (P_{in'}^a)^{\theta_{in'n}^{\iota}/(1-\theta_{in}^{\pi})}.$$

Compute the map between consumption and aggregate expenditures on sectoral composites following [\(H.35\)](#) and [\(H.36\)](#), setting

$$[\mathcal{X}^a]_{in,jn'} = \theta_{inn'}^Q \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a, \quad \text{where} \quad \zeta_{ijn}^a = \frac{(K_{ijn}/K_{jn})(p_{ijn}^a)^{1-\sigma}}{(P_{jn}^a)^{1-\sigma}}.$$

- iv. Solve for spending on sectoral composites given prices as follows.

- (a) At $b' = 1$, initialize all $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1,b'_{\text{final}}}$ if $a > 1$.⁶
- (b) Compute consumption expenditure according to [\(H.32\)](#), setting

$$C_{in}^{a,b'} = \theta_{in}^C P_i^{ca} \tilde{c}_i^{a,b'} + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^{\iota}}{1 - \theta_{in'}^{\pi}} P_{in'}^{\iota a} \tilde{c}_{in'}^{a,b'}$$

where $\tilde{c}_i^{a,b'} = (\psi_i^{a,b'} P_i^{ca})^{-\rho}$

$$\tilde{c}_{in}^{a,b'} = \left(\psi_i^{a,b'} P_{in}^{\iota a} / \tilde{\nu}_{in} \right)^{-1/\theta_{in}^{\pi}}$$

$$\tilde{\nu}_{in} = \nu_{in} \phi_{in}^k (1 - \theta_{in}^{\pi})^{\theta_{in}^{\pi}}.$$

⁶ Ψ_i is an auxiliary variable set in Supplemental Appendix [H.5](#).

(c) Compute sectoral composite spending $X_{in}^{a,b'}$ according to (H.36), setting $X^{a,b'} = [\text{id} - \mathcal{X}^a]^{-1} C^{a,b'}$.

(d) Compute trade tax revenues $T_i^{a,b'}$ according to (H.35) and (H.37), setting

$$T_i^{a,b'} = \sum_{o \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} \zeta_{oin}^a X_{in}^{a,b'} - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^{a,b'}.$$

(e) Compute households' trade-tax-inclusive incomes

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + T_i^{a,b'}.$$

(f) For some numerical parameter $\text{tol}_{\text{inner},2}$, test whether

$$\max_{i \in \mathcal{I}} \left| \log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} \right| < \text{tol}_{\text{inner},2}.$$

i. If not, update households' Lagrange multipliers according to

$$\psi_i^{a,b'+1} = \exp \left[\log \psi_i^{a,b'} + \text{step}_{\text{inner},2} \left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner},2} > 0$. Increase b' by one and return to step 4.b.

ii. If so, set $X_{in}^a \equiv X_{in}^{a,b'}$ and proceed to step 5.

v. Define $\tilde{w}_i^a$ and $\tilde{r}_{in}^a$ using (H.39), setting

$$\tilde{w}_i^a = \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a, \quad \tilde{r}_{in}^a = \frac{1}{k_{in}} \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a, \quad Y_{ijn}^a = \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a.$$

vi. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\max_{i \in \mathcal{I}} |\log \tilde{w}_i^a - \log w_i^a| < \text{tol}_{\text{outer}} \quad \text{and} \quad \max_{i \in \mathcal{I}, n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| < \text{tol}_{\text{outer}}.$$

(a) If not, update factor prices exactly as in step 5.a of Supplemental Appendix H.4.3, increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} \equiv r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{jn}^a$, and compute r_{ijn} using (H.40).

H.4.6 Numerical algorithm for equilibrium in conflict static model given Home trade

In the static model that represents the conflict period, capital is exogenous. We now describe the series of equilibrium conditions that determine all endogenous variables, given capital, Home's import quantities $\{m_{o \neq H, H, n}\}$ and export quantities $\{m_{H, d \neq H, n}\}$, and solve for the equilibrium in the rest of the world. Goods prices and consumer demand are given by (H.31) and (H.32) exactly as in Supplemental Appendix H.4.5.

Spending. The accounting identities (H.33) and (H.34) continue to hold. For flows that do not involve Home, $X_{ijn} = \zeta_{ijn} X_{jn}$ with ζ as in (H.35). For the Home country, by contrast,

$$X_{ijn} = p_{ijn} m_{ijn} \quad \text{for } i = H \text{ or } j = H. \quad (\text{H.41})$$

Combining these implies a linear system for the sectoral composite spending of the non-Home countries,

$$X_{-H} = [\text{id} - \mathcal{X}_{-H}]^{-1} X_{-H0} \quad (\text{H.42})$$

where $[X_{-H}]_{in} = X_{in}$

$$\begin{aligned} [X_{-H0}]_{in} &= C_{in} + \sum_{n' \in \mathcal{N}} \theta_{inn'}^Q \frac{1 + \tau_{iHn'}^x}{1 + \tau_{iHn'}^m} p_{iHn'} m_{iHn'} \\ [\mathcal{X}_{-H}]_{in, jn'} &= \theta_{inn'}^Q \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}. \end{aligned}$$

Lagrange multipliers on household budget constraints. Each country i has tax revenues given by (H.37), with the flows involving Home valued at (H.41). The Lagrange multiplier on each non-Home household budget constraint must be such that

$$\begin{aligned} \sum_{n \in \mathcal{N}} C_{in} &= w_i \ell_i + \sum_{n \in \mathcal{N}} r_{in} k_{in} + T_i + \Gamma_i \\ \text{where } \Gamma_i &= \frac{1}{|\mathcal{I}| - 1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}}{1 + \tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}}{1 + \tau_{oHn}^m} m_{oHn} \right]. \end{aligned} \quad (\text{H.43})$$

Home's own budget constraint is not imposed, since Home's trade vector is given; none of the values computed for $i = H$ are used.

Factor markets. Factor demand and market clearing are as in (H.39), restricted to $i \neq H$. The implicit rental rate on relationship capital is given by (H.40).

Numerical algorithm. We iterate over wages, production capital rental rates, Home export prices, sectoral composite prices, and Lagrange multipliers on household budget constraints in one outer and two successive inner loops indexed by a , b , and b' . The structure is identical to Supplemental Appendix H.4.5 with the following modifications.

- i. At $a = 1$, initialize all $w_i^{a=1} = 1$, $r_{in}^{a=1} = 1$, and $p_{Hjn}^{a=1} = 1$ for all $i, j \neq H$.
- ii. Solve for sectoral composite prices given factor prices as follows.
 - (a) For $i \neq H$, at $b = 1$, initialize all $P_{in}^{a,b=1} = 1$ if $a = 1$ or $P_{in}^{a,b=1} = P_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.
 - (b) Define $\tilde{P}_{in}^{a,b}$ for $i \neq H$ using (H.31), setting

$$p_{ijn}^{a,b} = \frac{1 + \tau_{ijn}^m}{1 + \tau_{ijn}^x} (\phi_{in}^q)^{-1} (w_i^a)^{\theta_{in}^l} (r_{in}^a)^{\theta_{in}^k} \prod_{n' \in \mathcal{N}} (P_{in'}^{a,b})^{\theta_{in'n}^Q}$$

$$\tilde{P}_{in}^{a,b} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (K_{oin}/K_{in}) (p_{oin}^{a,b})^{1-\sigma} \right]^{1/(1-\sigma)}$$

where $K_{in} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}.$

- (c) For some numerical parameter $\text{tol}_{\text{inner},1}$, test whether

$$\max_{i \neq H, n \in \mathcal{N}} |\log \tilde{P}_{in}^{a,b} - \log P_{in}^{a,b}| < \text{tol}_{\text{inner},1}.$$

- i. If not, update prices for $i \neq H$ according to

$$P_{in}^{a,b+1} = \exp \left[\log P_{in}^{a,b} + \text{step}_{\text{inner},1} \left(\log \tilde{P}_{in}^{a,b} - \log P_{in}^{a,b} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner},1} > 0$. Increase b by one and return to step 2.b.

- ii. If so, set $P_{in}^a \equiv P_{in}^{a,b}$, $p_{ijn}^a \equiv p_{ijn}^{a,b}$, for $i \neq H$, and proceed to step 3.

- iii. Compute true-consumption and investment-as-consumption price indices P_i^{ca} and $P_{in}^{\iota a}$ for $i \neq H$ according to (H.32), setting

$$P_i^{ca} = \prod_{n \in \mathcal{N}} (P_{in}^a)^{\theta_{in}^C}$$

$$P_{in}^{\iota a} = \prod_{n' \in \mathcal{N}} (P_{in'}^a)^{\theta_{in'n}^{\iota}/(1-\theta_{in}^{\pi})}.$$

Compute the map between consumption and aggregate expenditures on sectoral composites by non-Home countries following (H.42), setting

$$[\mathcal{X}_{-H}^a]_{in,jn'} = \theta_{inn'}^Q \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \zeta_{ijn'}^a$$

$$\text{where } \zeta_{ijn}^a = \frac{(K_{ijn}/K_{jn})(p_{ijn}^a)^{1-\sigma}}{(P_{jn}^a)^{1-\sigma}}$$

for $i, j \neq H$. Compute international transfers Γ_i^a following (H.43):

$$\Gamma_i^a = \frac{1}{|\mathcal{I}|-1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}^a}{1 + \tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}^a}{1 + \tau_{oHn}^m} m_{oHn} \right].$$

iv. Solve for spending on sectoral composites given prices as follows.

- (a) At $b' = 1$, for all $i \neq H$, initialize $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1,b'_{\text{final}}}$ if $a > 1$.⁷
- (b) Compute consumption expenditure for $i \neq H$ according to (H.32), setting

$$C_{in}^{a,b'} = \theta_{in}^C P_i^{ca} \tilde{c}_i^{a,b'} + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^t}{1 - \theta_{in'}^\pi} P_{in'}^{ia} \tilde{c}_{in'}^{a,b'}$$

$$\text{where } \tilde{c}_i^{a,b'} = (\psi_i^{a,b'} P_i^{ca})^{-\rho}$$

$$\tilde{c}_{in}^{a,b'} = \left(\psi_i^{a,b'} P_{in}^{ia} / \tilde{\nu}_{in} \right)^{-1/\theta_{in}^\pi}$$

$$\tilde{\nu}_{in} = \nu_{in} \phi_{in}^k (1 - \theta_{in}^\pi)^{\theta_{in}^\pi}.$$

- (c) Compute sectoral composite spending $X_{in}^{a,b'}$ for $i \neq H$ according to (H.42), setting

$$X_{-H}^{a,b'} = [\text{id} - \mathcal{X}_{-H}^a]^{-1} X_{-H0}^{a,b'}$$

$$\text{where } [X_{-H}^{a,b'}]_{in} = X_{in}^{a,b'}$$

$$[X_{-H0}^{a,b'}]_{in} = C_{in}^{a,b'} + \sum_{n' \in \mathcal{N}} \theta_{inn'}^Q \frac{1 + \tau_{iHn'}^x}{1 + \tau_{iHn'}^m} p_{iHn'}^a m_{iHn'}.$$

⁷ Ψ_i is an auxiliary variable set in Supplemental Appendix H.5.

(d) Compute trade tax revenues $T_i^{a,b'}$ for $i \neq H$ according to (H.35) and (H.37), setting

$$T_i^{a,b'} = \sum_{n \in \mathcal{N}} \frac{\tau_{Hin}^m}{1 + \tau_{Hin}^m} p_{Hin}^a m_{Hin} + \sum_{o \neq H} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} \zeta_{oin}^a X_{in}^{a,b'} \\ - \sum_{n \in \mathcal{N}} \frac{\tau_{iHn}^x}{1 + \tau_{iHn}^x} p_{iHn}^a m_{iHn} - \sum_{j \neq H} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^x} \zeta_{ijn}^a X_{jn}^{a,b'}.$$

(e) Compute households' trade-tax-inclusive incomes for $i \neq H$ (as in (H.43))

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + T_i^{a,b'} + \Gamma_i^a.$$

(f) For some numerical parameter $\text{tol}_{\text{inner},2}$, test whether

$$\max_{i \neq H} |\log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'}| < \text{tol}_{\text{inner},2}.$$

i. If not, update households' Lagrange multipliers for $i \neq H$ according to

$$\psi_i^{a,b'+1} = \exp \left[\log \psi_i^{a,b'} + \text{step}_{\text{inner},2} \left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner},2} > 0$. Increase b' by one and return to step 4.b.

ii. If so, set $X_{in}^a \equiv X_{in}^{a,b'}$ for $i \neq H$ and proceed to step 5.

v. Define $\tilde{w}_i^a$ and $\tilde{r}_{in}^a$ for $i \neq H$ using (H.39), setting

$$\tilde{w}_i^a = \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \tilde{r}_{in}^a = \frac{1}{k_{in}} \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \text{where } Y_{ijn}^a = \begin{cases} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a & \text{for } j \neq H \\ \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} p_{iHn}^a m_{iHn} & \text{for } j = H \end{cases}.$$

Similarly, for $i \neq H$, set

$$\tilde{p}_{Hin}^a = \frac{1}{m_{Hin}} \zeta_{Hin}^a X_{in}^a.$$

Without loss of generality, set

$$\tilde{w}_H^a = \tilde{r}_{Hn}^a = \tilde{p}_{HHn}^a = 1.$$

vi. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\begin{aligned} \max_{i \neq H} |\log \tilde{w}_i^a - \log w_i^a| &< \text{tol}_{\text{outer}} \\ \max_{i \neq H, n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| &< \text{tol}_{\text{outer}} \\ \text{and } \max_{i \neq H, n \in \mathcal{N}} |\log \tilde{p}_{Hin}^a - \log p_{Hin}^a| &< \text{tol}_{\text{outer}}. \end{aligned}$$

(a) If not, update factor prices according to

$$\begin{aligned} w_i^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{w}_i^a \\ r_{in}^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{r}_{in}^a \\ p_{Hin}^{a+1} &= \frac{\xi}{\sum_{j \neq H} \hat{w}_j^a \ell_j + \sum_{j \neq H, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{p}_{Hin}^a \end{aligned}$$

where $\hat{w}_i^a = \exp [\log w_i^a + \text{step}_{\text{outer}} (\log \tilde{w}_i^a - \log w_i^a)]$
 $\hat{r}_{in}^a = \exp [\log r_{in}^a + \text{step}_{\text{outer}} (\log \tilde{r}_{in}^a - \log r_{in}^a)]$
 $\hat{p}_{Hin}^a = \exp [\log p_{Hin}^a + \text{step}_{\text{outer}} (\log \tilde{p}_{Hin}^a - \log p_{Hin}^a)]$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and, for $i \neq H$, set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{i \neq H, n} \equiv X_{in}^a$, $X_{i, j \neq H, n} \equiv \zeta_{ijn}^a X_{jn}^a$, $X_{iHn} \equiv p_{iHn}^a m_{iHn}$, and—using (H.40)—set

$$r_{ijn} = \frac{X_{jn}}{K_{ijn}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{ijn}^K > 0} \left(\frac{(K_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(P_{jn})^{1-\sigma}} - \frac{(\theta_{ijn}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right)$$

for $j \neq H$.

H.4.7 Numerical algorithm for equilibrium of conflict period given own trade quantities

We now fix a country i with import quantities $\{m_{o \neq i, i, n}\}$, export quantities $\{m_{i, d \neq i, n}\}$, production capital $\{k_{in}\}$, and relationship capital $\{K_{oin}\}$. We assume these imports and exports are simply an endowment of the representative consumer (a negative endowment in

the case of exports) and solve for the equilibrium in the conflict period taking them as given. This is the Meade utility function of country i , and it is the object we differentiate to build the manipulability term in Supplemental Appendix H.3.

Our algorithm iterates over wages, production capital rental rates, imported good prices, sectoral composite prices, and Lagrange multipliers on the household budget constraint in one outer and two successive inner loops indexed by a , b , and b' , respectively. Although variables are indexed by i , we compute them only for the fixed country i of interest.

- i. At $a = 1$, initialize all $w_i^{a=1}$, $r_{in}^{a=1}$, and $p_{o \neq i, in}^{a=1}$ at their supplied initial guesses. Prices on cells with no trade are set to infinity.
- ii. Solve for sectoral composite prices given factor and import prices as follows.
 - (a) At $b = 1$, initialize all $P_{in}^{a,b=1} = 1$ if $a = 1$ or $P_{in}^{a,b=1} = P_{in}^{a-1, b_{\text{final}}}$ if $a > 1$.
 - (b) Define $\tilde{P}_{in}^{a,b}$ using (H.31), setting

$$p_{iin}^{a,b} = (\phi_{in}^q)^{-1} (w_i^a)^{\theta_{in}^\ell} (r_{in}^a)^{\theta_{in}^k} \prod_{n' \in \mathcal{N}} (P_{in'}^{a,b})^{\theta_{in'}^Q}$$

$$\tilde{P}_{in}^{a,b} = \left[\mathbb{1}_{\theta_{iin}^K > 0} (K_{iin}/K_{in}) (p_{iin}^{a,b})^{1-\sigma} + \sum_{o \neq i: \theta_{oin}^K > 0} (K_{oin}/K_{in}) (p_{oin}^a)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

$$\text{where } K_{in} = \left[\sum_{o \in \mathcal{I}: \theta_{oin}^K > 0} (\theta_{oin}^K)^{\frac{1-\sigma}{\bar{\sigma}-\sigma}} (K_{oin})^{\frac{\bar{\sigma}-1}{\bar{\sigma}-\sigma}} \right]^{\frac{\bar{\sigma}-\sigma}{\bar{\sigma}-1}}.$$

- (c) For some numerical parameter $\text{tol}_{\text{inner},1}$, test whether

$$\max_{n \in \mathcal{N}} \left| \log \tilde{P}_{in}^{a,b} - \log P_{in}^{a,b} \right| < \text{tol}_{\text{inner},1}$$

and, if not, update prices log-linearly with step $\text{step}_{\text{inner},1}$ and return to step 2.b. If so, set $P_{in}^a \equiv P_{in}^{a,b}$, $p_{iin}^a \equiv p_{iin}^{a,b}$, and proceed to step 3.

- iii. Compute true-consumption and investment-as-consumption price indices P_i^{ca} and $P_{in}^{\iota a}$ according to (H.32), setting

$$P_i^{ca} = \prod_{n \in \mathcal{N}} (P_{in}^a)^{\theta_{in}^C}, \quad P_{in}^{\iota a} = \prod_{n' \in \mathcal{N}} (P_{in'}^a)^{\frac{\theta_{in'}^\iota p_{in'}^{\theta_{in'}^K}}{1-\theta_{in'}^K}}.$$

- iv. Solve for spending on sectoral composites given prices as follows.

- (a) At $b' = 1$, initialize $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1, b'_{\text{final}}}$ if $a > 1$.⁸

⁸ Ψ_i is an auxiliary variable set in Supplemental Appendix H.5.

(b) Compute consumption expenditure according to (H.32), setting

$$C_{in}^{a,b'} = \theta_{in}^C P_i^{ca} \tilde{c}_i^{a,b'} + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^c}{1 - \theta_{in'}^\pi} P_{in'}^{ia} \tilde{c}_{in'}^{a,b'}$$

where $\tilde{c}_i^{a,b'} = \left(\psi_i^{a,b'} P_i^{ca} \right)^{-\rho}$

$$\tilde{c}_{in}^{a,b'} = \left(\psi_i^{a,b'} P_{in}^{ia} / \tilde{\nu}_{in} \right)^{-1/\theta_{in}^\pi}$$

$$\tilde{\nu}_{in} = \nu_{in} \phi_{in}^k (1 - \theta_{in}^\pi)^{\theta_{in}^\pi}.$$

(c) Compute the representative household's income inclusive of the value of its endowment of imports less the cost of producing its negative endowment of exports,

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + \sum_{j \neq i} \sum_{n \in \mathcal{N}} [p_{jin}^a m_{jin} - m_{ijn} p_{iin}^a].$$

(d) For some numerical parameter $\text{tol}_{\text{inner},2} > 0$, test whether

$$\left| \log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} \right| < \text{tol}_{\text{inner},2}.$$

i. If not, update the household's Lagrange multiplier according to

$$\psi_i^{a,b'+1} = \exp \left[\log \psi_i^{a,b'} + \text{step}_{\text{inner},2} \left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner},2} > 0$. Increase b' by one and return to step 4.b.

ii. If so, set $C_{in}^a \equiv C_{in}^{a,b'}$ and proceed to step 5.

v. Compute sectoral composite spending X_{in}^a according to (H.33)-(H.35) and the observation that producing an export $m_{i,j \neq i,n}$ requires production costs of $p_{iin} m_{ijn}$ in sector n and

country i . Namely, set

$$\begin{aligned}
X^a &= [\text{id} - \mathcal{X}^a]^{-1} X_0^a \\
\text{where } [X^a]_{in} &= X_{in}^a \\
[X_0^a]_{in} &= C_{in}^a + \sum_{j \neq i} \sum_{n' \in \mathcal{N}} \theta_{inn'}^Q p_{iin'}^a m_{ijn'} \\
[\mathcal{X}^a]_{n,n'} &= \theta_{inn'}^Q \zeta_{iin'}^a \\
\zeta_{oin}^a &= \frac{(K_{oin}/K_{in}) (p_{oin}^a)^{1-\sigma}}{(P_{in}^a)^{1-\sigma}}.
\end{aligned}$$

vi. Define $\tilde{w}_i^a$, $\tilde{r}_{in}^a$, and $\tilde{p}_{oin}^a$ using (H.39), setting

$$\begin{aligned}
\tilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\
\tilde{r}_{in}^a &= \frac{1}{k_{in}} \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \\
\tilde{p}_{oin}^a &= \frac{1}{m_{oin}} \zeta_{oin}^a X_{in}^a \\
Y_{ijn}^a &= \begin{cases} \zeta_{iin}^a X_{in}^a & \text{if } i = j \\ p_{iin}^a m_{ijn} & \text{if } i \neq j \end{cases}.
\end{aligned}$$

vii. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\begin{aligned}
&|\log \tilde{w}_i^a - \log w_i^a| < \text{tol}_{\text{outer}} \\
&\max_{n \in \mathcal{N}} |\log \tilde{r}_{in}^a - \log r_{in}^a| < \text{tol}_{\text{outer}} \\
&\max_{o \neq i, n \in \mathcal{N}} |\log \tilde{p}_{oin}^a - \log p_{oin}^a| < \text{tol}_{\text{outer}}.
\end{aligned}$$

(a) If not, update factor and import prices according to

$$\begin{aligned}
w_i^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{w}_i^a \\
r_{in}^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{r}_{in}^a \\
p_{o \neq i, in}^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{p}_{oin}^a \\
\hat{w}_i^a &= \exp [\log w_i^a + \text{step}_{\text{outer}} (\log \tilde{w}_i^a - \log w_i^a)] \\
\hat{r}_{in}^a &= \exp [\log r_{in}^a + \text{step}_{\text{outer}} (\log \tilde{r}_{in}^a - \log r_{in}^a)] \\
\hat{p}_{oin}^a &= \exp [\log p_{oin}^a + \text{step}_{\text{outer}} (\log \tilde{p}_{oin}^a - \log p_{oin}^a)]
\end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} \equiv r_{in}^a$, $P_{in} \equiv P_{in}^a$, $p_{o \neq i, in}^w \equiv p_{oin}^a$, $p_{i, j \neq i, n}^w \equiv p_{iin}^a$, $X_{in} \equiv X_{in}^a$, $C_{in} \equiv C_{in}^a$, and, using (H.40),

$$r_{oin} = \frac{X_{in}}{K_{oin}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{oin}^K > 0} \left(\frac{(K_{oin}/K_{in}) (p_{oin})^{1-\sigma}}{(P_{in})^{1-\sigma}} - \frac{(\theta_{oin}^K)^{(1-\sigma)/(\bar{\sigma}-\sigma)} (K_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{in})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right).$$

Note that the world prices implicitly assume country i sets its trade taxes to zero.

H.5 Model Calibration

This appendix details the model calibration procedure outlined in Section 6.2. The calibration takes as inputs cleaned 2017 data on and imputations of tariffs $\tau_{ijn}^{m, data}$ and export subsidies $\tau_{ijn}^{x, data}$; trade shares ζ_{ijn}^{data} ; labor, capital, and intermediate shares of gross output, $\theta_{in}^{\ell, data}$, $\theta_{in}^{k, data}$, and $\theta_{in'n}^{Q, data}$; directed investment shares of capital income, $\theta_{in'n}^{\iota, data}$; consumption shares $\theta_{in}^{C, data}$; and final consumption expenditures C_i^{data} . See Supplemental Appendices G.1 and G.2 for a description of how we construct these variables.

We calibrate the elasticities $\{\sigma, \bar{\sigma}, \rho\}$, preference and production shifters $\{\theta_{in}^C, \nu_{in}, \theta_{in}^{\ell}, \theta_{in}^k, \theta_{in'n}^Q, \theta_{in'n}^{\iota}, \theta_{ijn}^K, \phi_{in}^q, \phi_{in}^k\}$, and labor endowments $\{\ell_j\}$ under two assumptions. First, the data we observe correspond to the peacetime period in the $\beta \rightarrow 0$ limit. This implies that the static equilibrium conditions described in Supplemental Appendix H.4.2 apply. Second, under free trade, the economy is stationary in the sense that sectoral composite prices are the same at both times, up to a constant, and time-1 consumption and investment spending coincide with time-2 consumption-as-consumption and investment-as-consumption spending, up to the same constant.

Our calibration normalizes the units of account of origin-sector goods, labor, and production capital in each country so that $p_{in} = w_i = r_{in} = 1$ in the observed peacetime equilibrium for all $i \in \mathcal{I}$ and $n \in \mathcal{N}$. We then calibrate parameters as follows.

In addition to the parameters calibrated below, our calibration saves the data-implied level of nominal world GDP, ξ , and Lagrange multipliers on household budget constraints Ψ_i . Our simulations use ξ to normalize prices so that nominal world GDP is held constant. They use Ψ_i to form a starting guess for the Lagrange multiplier on households' budget constraint in the conflict period.

Elasticities. We set σ and $\bar{\sigma}$ to match the evidence on short- and long-run trade elasticities from [Boehm et al. \(2023\)](#). Following [Boehm et al. \(2024\)](#), we set the short-run elasticity to $\sigma = 1.25$ and the long-run elasticity to $\bar{\sigma} = 2$. We set $\rho = 0.53$, the mean estimate for China's elasticity of intertemporal substitution reported in the meta-analysis of [Havranek et al. \(2015\)](#).

Relationship capital productivity. Since $p_{in} = 1$, the price of variety in in country j is simply the tax, $p_{ijn}^{data} = \frac{1+\tau_{ijn}^{m,data}}{1+\tau_{ijn}^{x,data}}$. Then using [\(H.9\)](#) and [\(H.15\)](#), the relationship capital productivity parameters are

$$\theta_{ijn}^K = \frac{(p_{ijn}^{data})^{\bar{\sigma}-1} \zeta_{ijn}^{data}}{\sum_{i'} (p_{i'jn}^{data})^{\bar{\sigma}-1} \zeta_{i'jn}^{data}}.$$

Preference and production shares. We set all production and preference share parameters to their empirical analogs in the 2017 data. Concretely, we set

$$\begin{aligned} \theta_{in}^C &= \theta_{in}^{C,data}, & \theta_{in}^\ell &= \theta_{in}^{\ell,data}, & \theta_{in}^k &= \theta_{in}^{k,data}, \\ \theta_{in'n}^Q &= \theta_{in'n}^{Q,data}, & \theta_{in'n}^\iota &= \theta_{in'n}^{\iota,data}. \end{aligned}$$

Labor endowments. We set countries' labor endowments to the levels required to produce the final consumption vector we observe in the data. To this end, we first compute the vector of country-sector gross outputs using the following expression for revenues in terms of consumption, based on [\(H.14\)](#), the price normalization, [\(H.15\)](#), [\(H.13\)](#), and [\(H.11\)](#):

$$Y_{in} = \sum_{j \in \mathcal{I}} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn} \left(\theta_{jn}^C C_j + \sum_{n' \in \mathcal{N}} \left(\theta_{jnn'}^Q + \theta_{jn'}^k \theta_{jnn'}^\iota \right) Y_{jn'} \right).$$

Inverting this expression implies a matrix expression for Y_{in} :

$$Y = (\text{id} - \mathcal{M})^{-1} \tilde{C}$$

where $[\mathcal{M}]_{in,jn'} \equiv \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn} \left(\theta_{jnn'}^Q + \theta_{jn'}^k \theta_{jnn'}^\ell \right)$

$$[\tilde{C}]_{in} \equiv \sum_{j \in \mathcal{I}} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn} \theta_{jn}^C C_j.$$

We define Y_{in}^{data} by

$$Y^{data} \equiv (\text{id} - \mathcal{M})^{-1} \tilde{C},$$

where $\mathcal{M}$ and $\tilde{C}$ are computed using $\tau_{ijn}^x = \tau_{ijn}^{x,data}$, $\tau_{ijn}^m = \tau_{ijn}^{m,data}$, $\zeta_{ijn} = \zeta_{ijn}^{data}$, and $C_j = C_j^{data}$. Finally, we set labor endowments to meet the labor demand implied by this level of gross output:

$$\ell_i = \sum_{n \in \mathcal{N}} \theta_{in}^\ell Y_{in}^{data}.$$

Nominal world GDP. We compute nominal world GDP as

$$\xi = \sum_{i \in \mathcal{I}, n \in \mathcal{N}} (\theta_{in}^\ell + \theta_{in}^k) Y_{in}^{data}.$$

Production shifters. We set the production shifters ϕ_{in}^q to be consistent with our price normalization. Composite producer cost minimization implies

$$P_{jn'}^{data} = \left[\sum_i \theta_{ijn'}^K (p_{ijn'}^{data})^{1-\bar{\sigma}} \right]^{\frac{1}{1-\bar{\sigma}}}.$$

Then since the price of varieties is one, (H.9) implies

$$\phi_{in}^q = \prod_{n'} (P_{in'}^{data})^{\theta_{in'}^Q},$$

for all $i \in \mathcal{I}$ and $n \in \mathcal{N}$.

To calibrate ϕ_{in}^k we first compute the level of capital bills R_{in}^{data} implied by the revenues Y_{in}^{data} consistent with the rest of our calibration, namely

$$R_{in}^{data} \equiv \theta_{in}^k \sum_{j \in \mathcal{I}} \frac{1 + \tau_{ijn}^{x,data}}{1 + \tau_{ijn}^{m,data}} \zeta_{ijn}^{data} \left(\theta_{jn}^C C_j^{data} + \sum_{n' \in \mathcal{N}} \left(\theta_{jnn'}^Q + \theta_{jn'}^k \theta_{jnn'}^\ell \right) Y_{jn'}^{data} \right).$$

The price normalization and (H.20) together imply that

$$\phi_{in}^k = (R_{in}^{data})^{\theta_{in}^\pi} \prod_{n' \in \mathcal{N}} (P_{in'}^{data})^{\theta_{in'n}^\iota}.$$

Investment-as-consumption preferences. Recall we assume that, under free trade, sectoral composite prices are the same at both times, up to a constant, and time-1 consumption and investment spending coincide with time-2 consumption-as-consumption and investment-as-consumption spending, up to the same constant.

Under our time-invariance assumption, (H.32) implies

$$\begin{aligned} \tilde{\nu}_{in} &= \frac{(P_{in}^{\iota, data})^{1-\theta_{in}^\pi} ((1-\theta_{in}^\pi) R_{in}^{data})^{\theta_{in}^\pi}}{(P_i^{c, data})^{(\rho-1)/\rho} (C_i^{data})^{1/\rho}} \\ \text{where } P_i^{c, data} &\equiv \prod_{n \in \mathcal{N}} (P_{in}^{data})^{\theta_{in}^C} \\ P_{in}^{\iota, data} &\equiv \prod_{n' \in \mathcal{N}} (P_{in'}^{data})^{\theta_{in'n}^\iota / (1-\theta_{in}^\pi)} \end{aligned}$$

for $\tilde{\nu}_{in} \equiv \nu_{in} \phi_{in}^k (1 - \theta_{in}^\pi)^{\theta_{in}^\pi}$. We accordingly set

$$\nu_{in} \equiv \frac{\tilde{\nu}_{in}}{\phi_{in}^k (1 - \theta_{in}^\pi)^{\theta_{in}^\pi}}.$$

Lagrange multipliers on household budget constraints. Using (H.32), we set Ψ_i to the implied Lagrange multiplier on the household budget constraint, i.e.,

$$\Psi_i \equiv \left(\frac{C_i^{data}}{(P_i^{c, data})^{1-\rho}} \right)^{-1/\rho}.$$